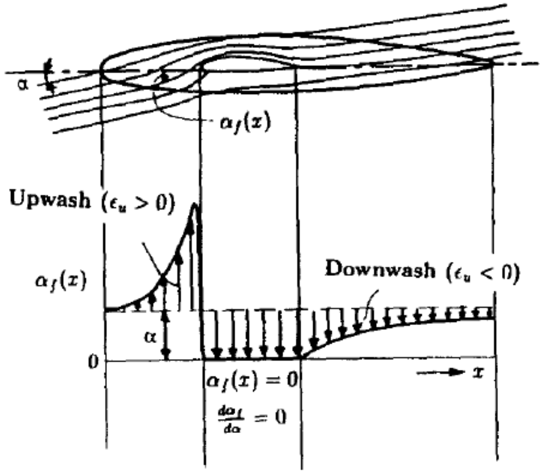
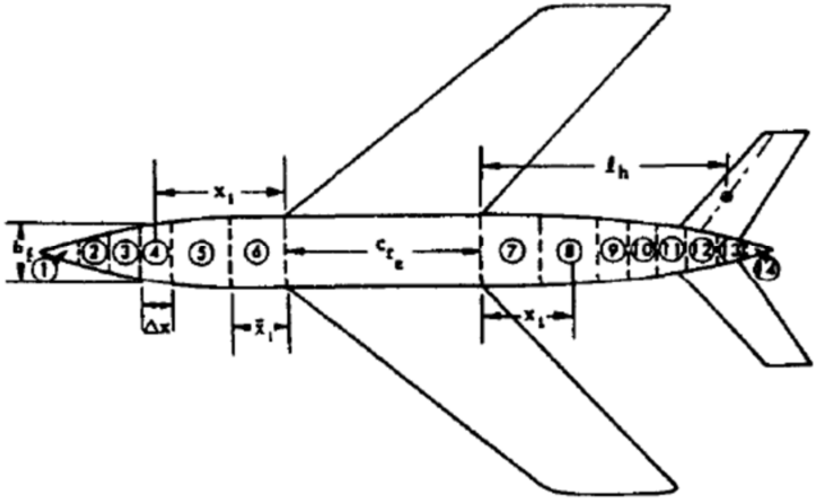


Fig. 3.7 Schematic diagram of the fuselage flow field in the presence of the wing.

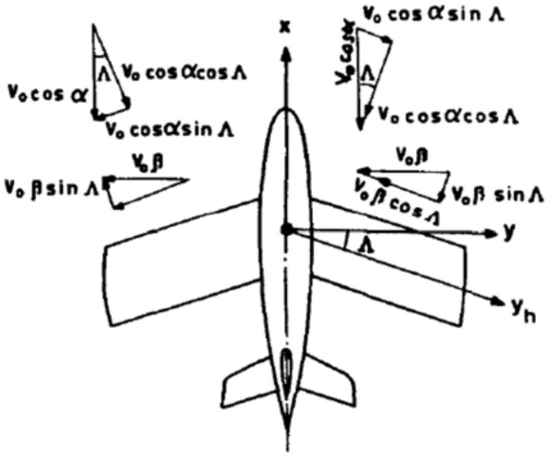


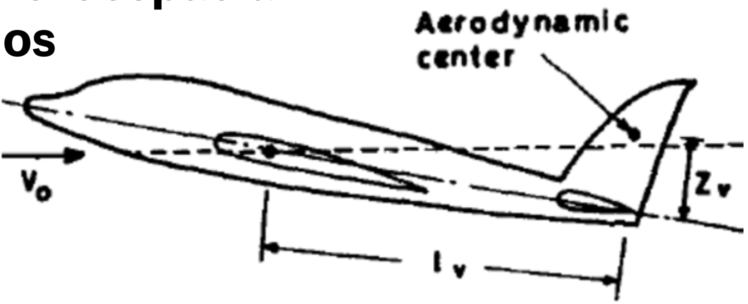
Estabilidad y Control Detallado

Derivadas Estabilidad Longitudinal

Tema 14.2

Sergio Esteban Roncero
Departamento de Ingeniería Aeroespacial
Y Mecánica de Fluidos





Derivadas $C_{D_\alpha}, C_{L_\alpha}, C_{M_\alpha}$

Angle of Attack Derivatives

Estimación Derivadas

- Contribución C_{D_α}
-
- Contribución C_{L_α}
    - Ala
    - Canard/horizontal/V-tail
    - Fuselaje
- Contribución C_{M_α}
    - Ala
    - Canard/horizontal/V-tail
    - Fuselaje

Derivadas en 1/rad si no se indica lo contrario
Si las derivadas no están en 1/rad hay que convertirlas

$C_{D\alpha}$

Estimación C_{D_α}

$$C_L = C_{L\alpha}\alpha$$

$$C_D = C_{DO} + kC_L^2,$$

Asumiendo modelo polar no compensada

$$C_{D\alpha} = \left(\frac{dC_D}{dC_L}\right)\left(\frac{dC_L}{d\alpha}\right) = 2kC_L C_{L\alpha}$$

$$k = 1/\pi Ae.$$

Coeficiente de resistencia inducida

Coeficiente de Oswald (dept. aerodinámica) →

$$e = \frac{1.1C_{L\alpha}}{RC_{L\alpha} + (1-R)\pi A}$$

→

$$R = a_1\lambda_1^3 + a_2\lambda_1^2 + a_3\lambda_1 + a_4$$

$$a_1 = 0.0004,\ \ a_2 = -0.0080,\ \ a_3 = 0.0501,\ \ a_4 = 0.8642,$$

$$\lambda_1 = A\lambda / \cos\Lambda_{LE}$$

A is the aspect ratio, λ is the taper ratio, and Λ_{LE} is the leading-edge sweep of the wing.

C_{L_α} of the entire Airplane

$$\begin{aligned}\Sigma F_x &= W - L = \frac{W}{qS} - C_{L_0} - C_{L_\alpha}\alpha - C_{L_{\delta_e}}\delta_e\\ \Sigma M &= 0 = C_{M_0} + C_{M_\alpha}\alpha + C_{M_{\delta_e}}\delta_e\end{aligned}$$

$$\begin{aligned}C_{L_{\delta_e}} &= C_{L_{\delta_c}} + C_{L_{\delta_t}}\\ C_{L_{\delta_c}} &= \frac{q_c}{q}\frac{S_c}{S}C_{L_{\delta_{c_{\delta_e}}}}\\ C_{L_{\delta_t}} &= \frac{q_t}{q}\frac{S_t}{S}C_{L_{\delta_{t_{\delta_e}}}}\end{aligned}$$

Efectividad de las Superficies de control

$$C_{L_0} = C_{L_{0_{WB}}} + \frac{q_c}{q}\frac{S_c}{S}C_{L_{0_c}} + \frac{q_t}{q}\frac{S_t}{S}C_{L_{0_t}} + C_{L_{\alpha_{WB}}}i_w + \frac{q_c}{q}\frac{S_c}{S}C_{L_{\alpha_c}}(i_c + \varepsilon_{0_c}) + \frac{q_t}{q}\frac{S_t}{S}C_{L_{\alpha_t}}(i_t - \varepsilon_{0_t})$$

$$C_{L_\alpha} = C_{L_{\alpha_{WB}}} + \frac{q_c}{q}\frac{S_c}{S}C_{L_{\alpha_c}}\left(1 + \frac{\partial\varepsilon_c}{\partial\alpha}\right) + \frac{q_t}{q}\frac{S_t}{S}C_{L_{\alpha_t}}\left(1 - \frac{\partial\varepsilon_t}{\partial\alpha}\right)$$

$C_{L_{\alpha_{WB}}}$ → representa la pendiente de sustentación del conjunto ala-fuselaje

En 1ª hipótesis sólo las superficies aerodinámicas generan sustentación

En 2ª hipótesis se puede estimar la contribución del fuselaje $\left(C_{L_{\alpha_{fus}}}\ y C_{M_{\alpha_{fus}}}\right)$

- Mediante métodos experimentales : análisis software XFLR5
- Mediante métodos empíricos: ecuaciones analíticas función de geometría

$$C_{L_{\alpha_{WB}}} = C_{L_{\alpha_W}} + C_{L_{\alpha_f}}$$

$C_{L\alpha,w}$, $C_{L\alpha,t}$ and $C_{L\alpha,c}$

Cálculo de C_{L_α} para cualquier superficie aerodinámica

Se emplean los métodos ya descritos en Aerodinámica

- Métodos experimentales (XFLR5)
- Métodos analíticos

$$a_w = \frac{2\pi A}{2+\sqrt{\frac{A^2\beta^2}{k^2}\left(1+\frac{\tan^2\Lambda_{c/2}}{\beta^2}\right)+4}}$$

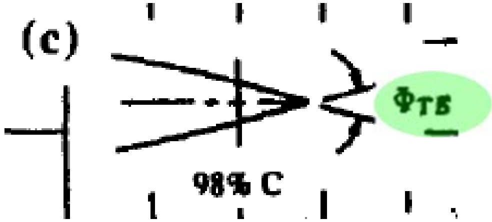
$$k = a_o/2\pi \qquad \beta = \sqrt{1-M^2},$$

$\Lambda_{c/2}$ is the midchord sweep.

a_o The sectional (two-dimensional) lift-curve slope a_o

$$a_o = \frac{1.05}{\sqrt{1-M^2}}\left[\frac{a_o}{(a_o)_{\text{theory}}}\right](a_o)_{\text{theory}}$$

$$\tan\frac{\phi'_{TE}}{2} = \frac{0.5y_{90}-0.5y_{99}}{9}$$



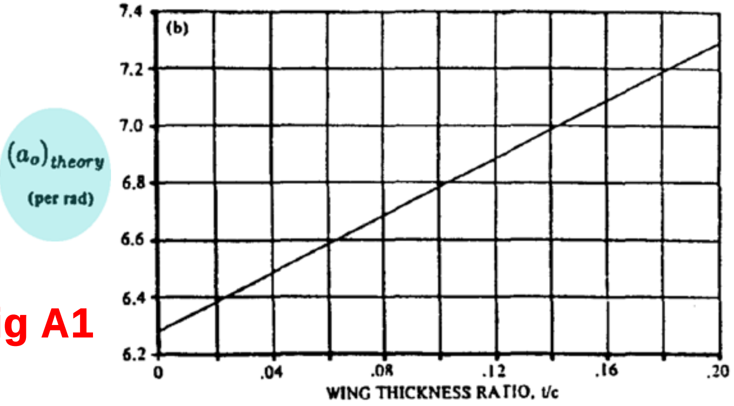


Fig A1

a) Theoretical sectional lift-curve slope

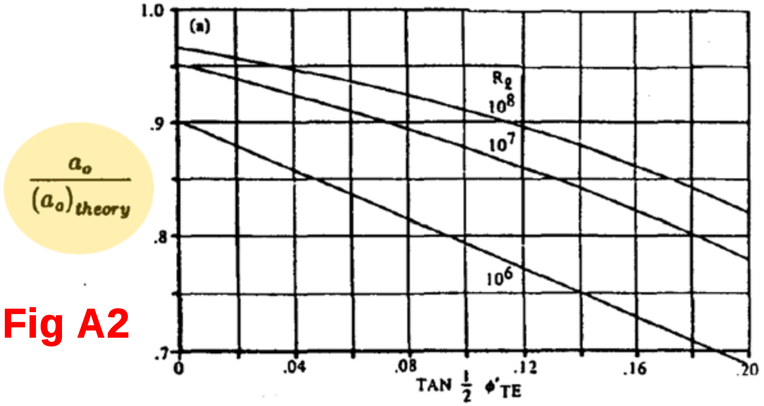


Fig A2

b) Empirical correction factor

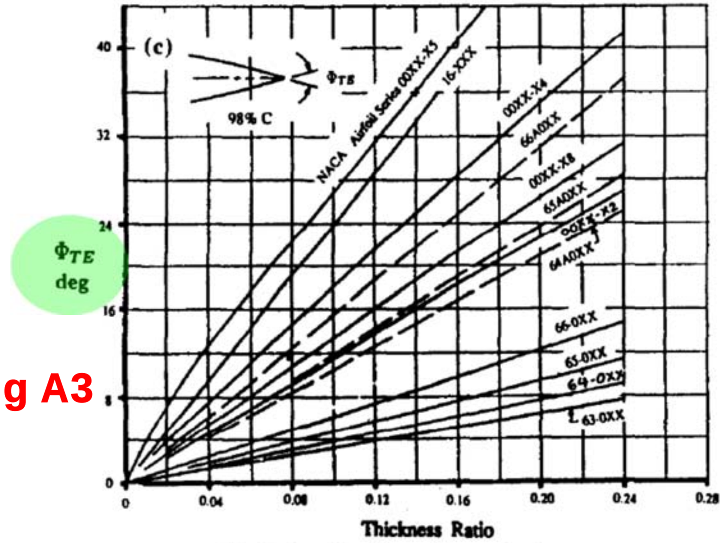
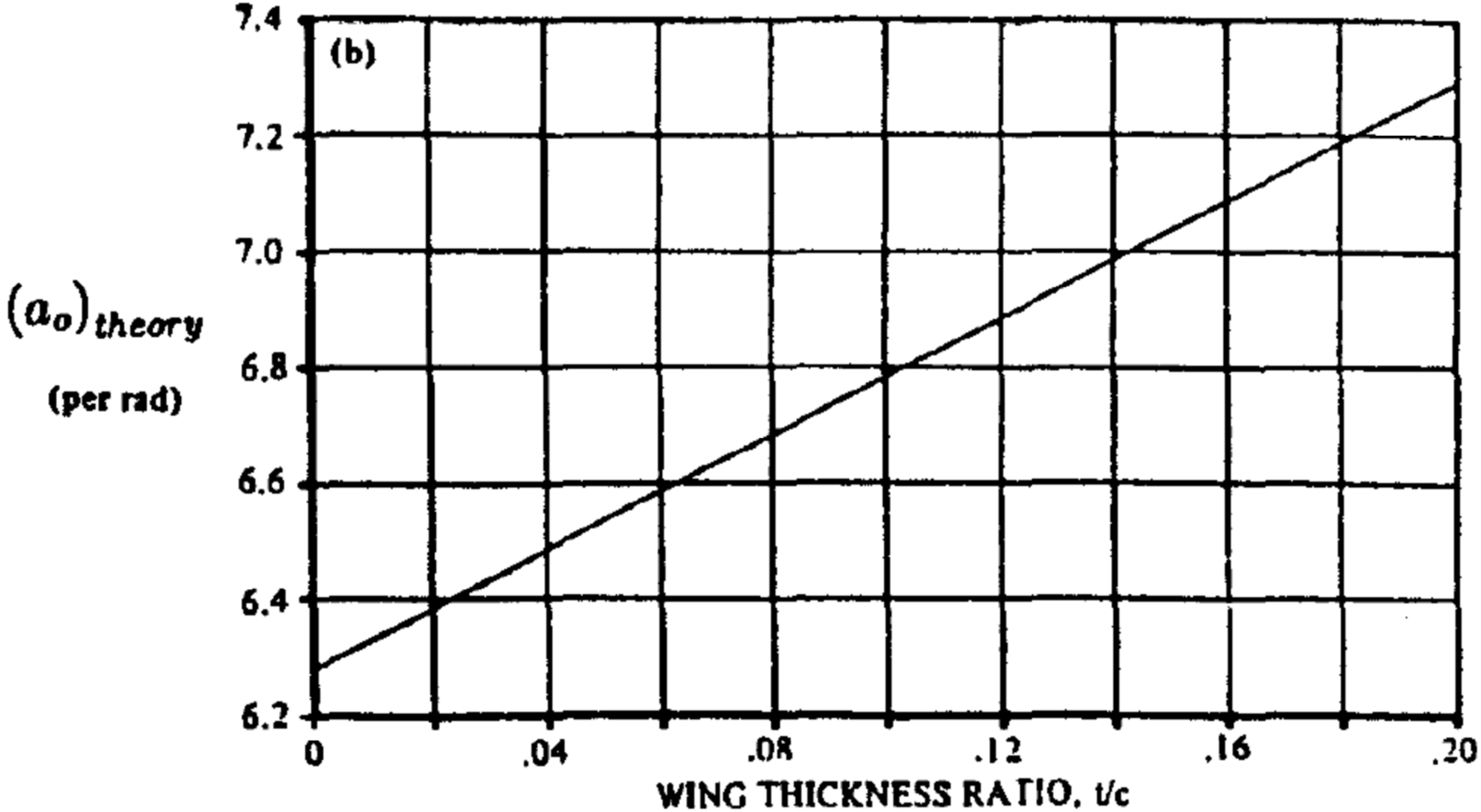


Fig A3

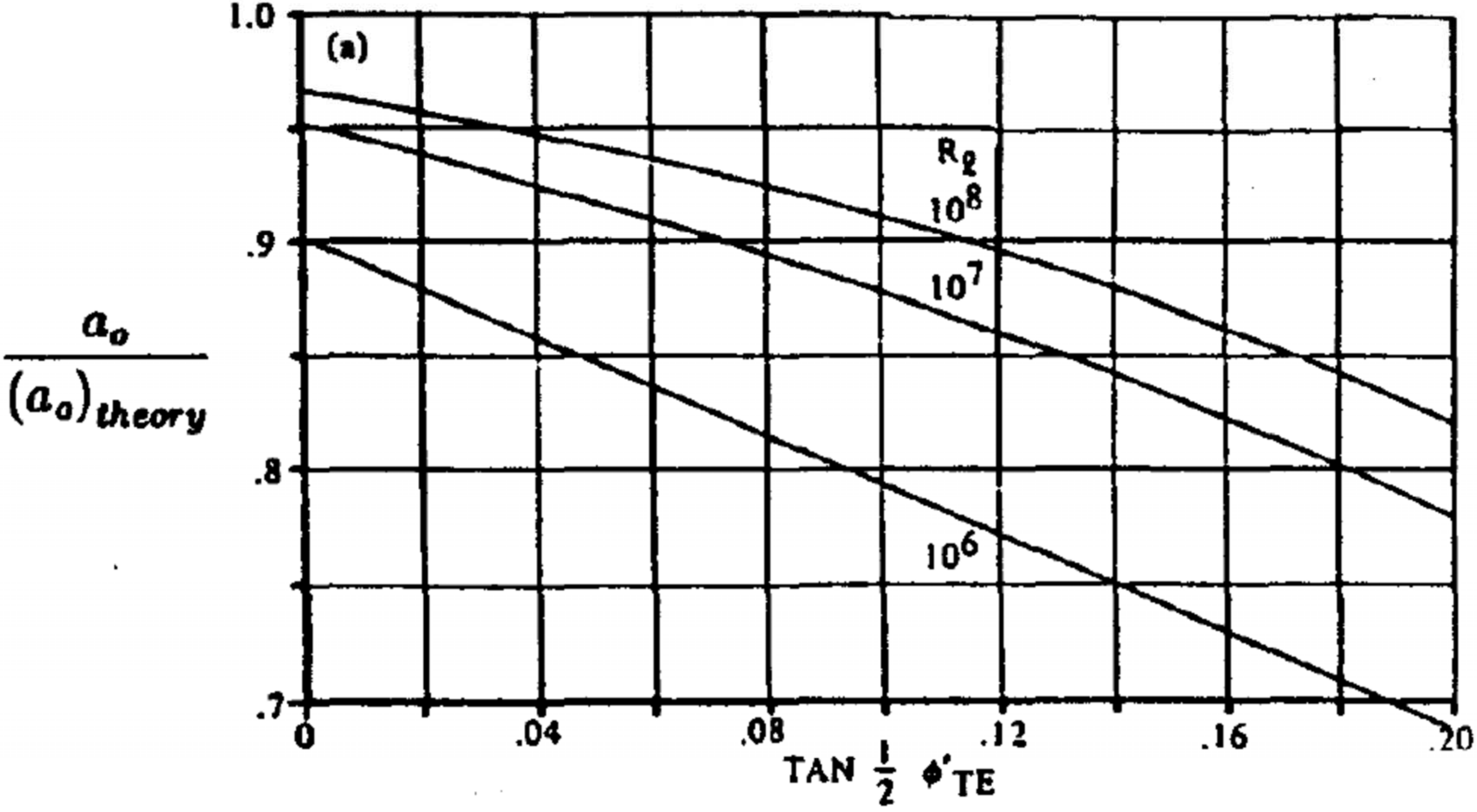
c) Variation of trailing-edge angle ϕ_{TE}

Fig A1



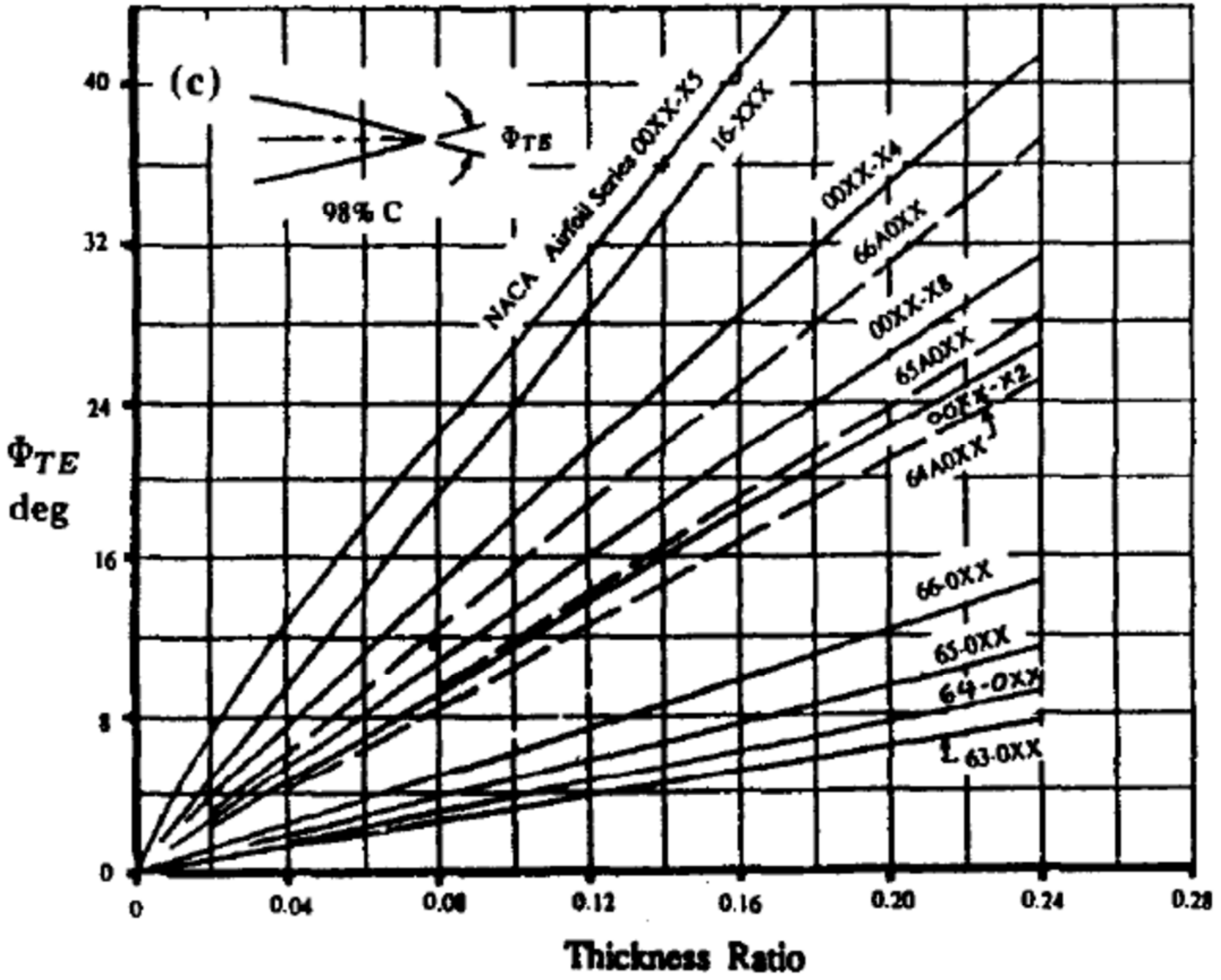
a) Theoretical sectional lift-curve slope

Fig A2



b) Empirical correction factor

Fig A3



c) Variation of trailing-edge angle ϕ_{TE}

Fig. 3.13 Sectional (two-dimensional) lift-curve slope of wings, continued.[1]

$C_{L_{\alpha_f}}$

$$C_{L_{\alpha_{Body}}} = \frac{2(k_2 - k_1)\, S_0}{V^{2/3}}$$

k_2 – k_1 is the apparent mass constant which is a function of fineness ratio (length/maximum thickness)

V_b = total body volume

S_0 = cross sectional area at x_0.

x_0. = body station where flow ceases to be potential, this is a function of x_1, the body station where the parameter dS_x/dx first reaches its minimum value. (This station where the change in area with respect to x first reaches its lowest value can be estimated from a sketch of the body.)

S_x = body cross sectional area at any body station

l_b = body length.

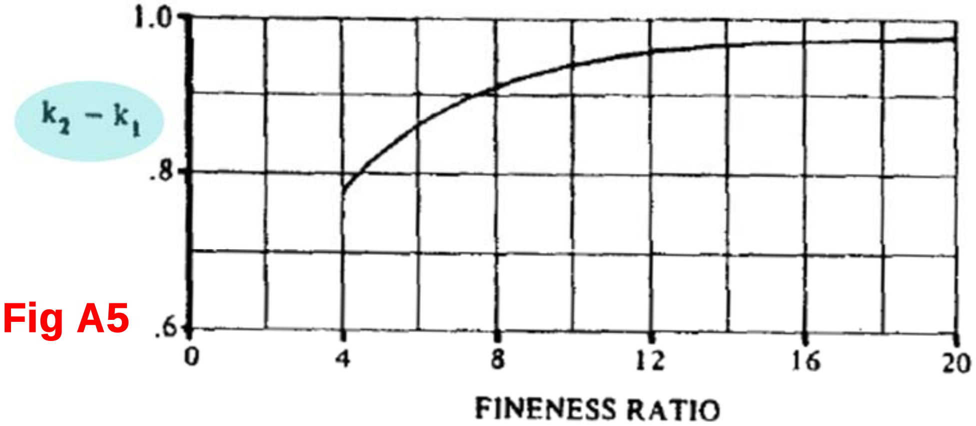


Fig A5

Fig. 3.6 Fuselage apparent mass coefficient.[1]

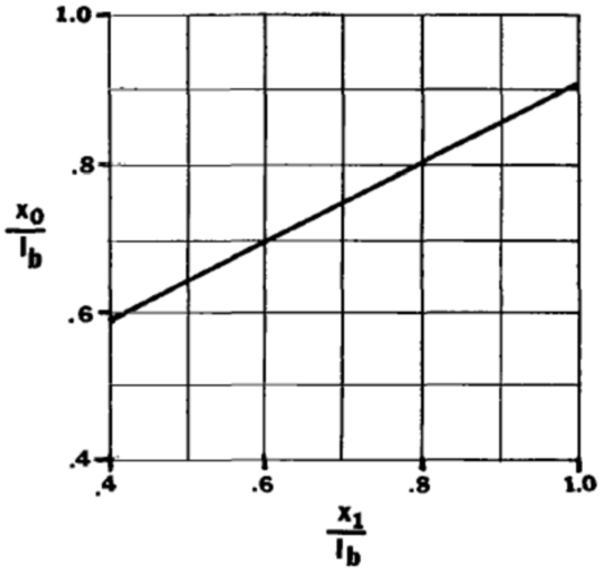
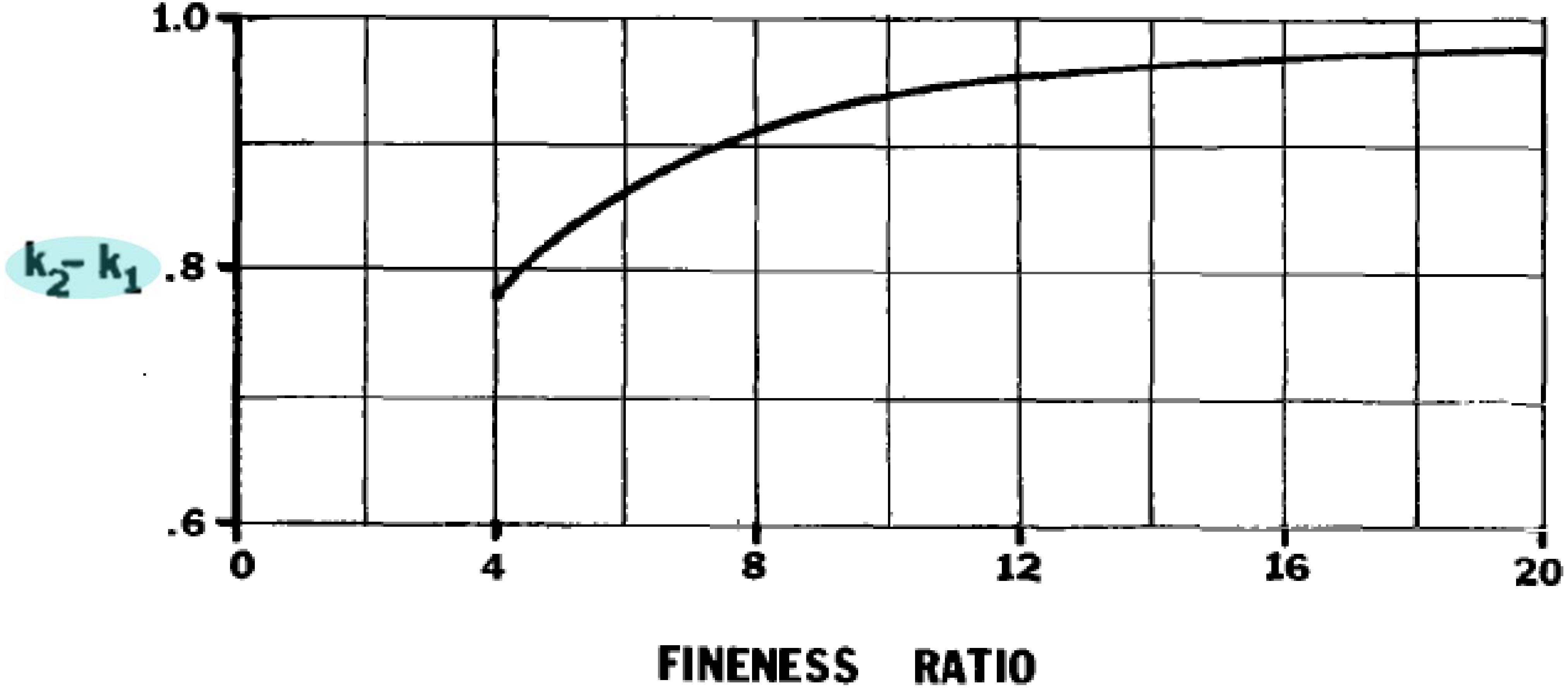


Fig A6

Figure 7. Body station where flow becomes viscous.

Fig A5

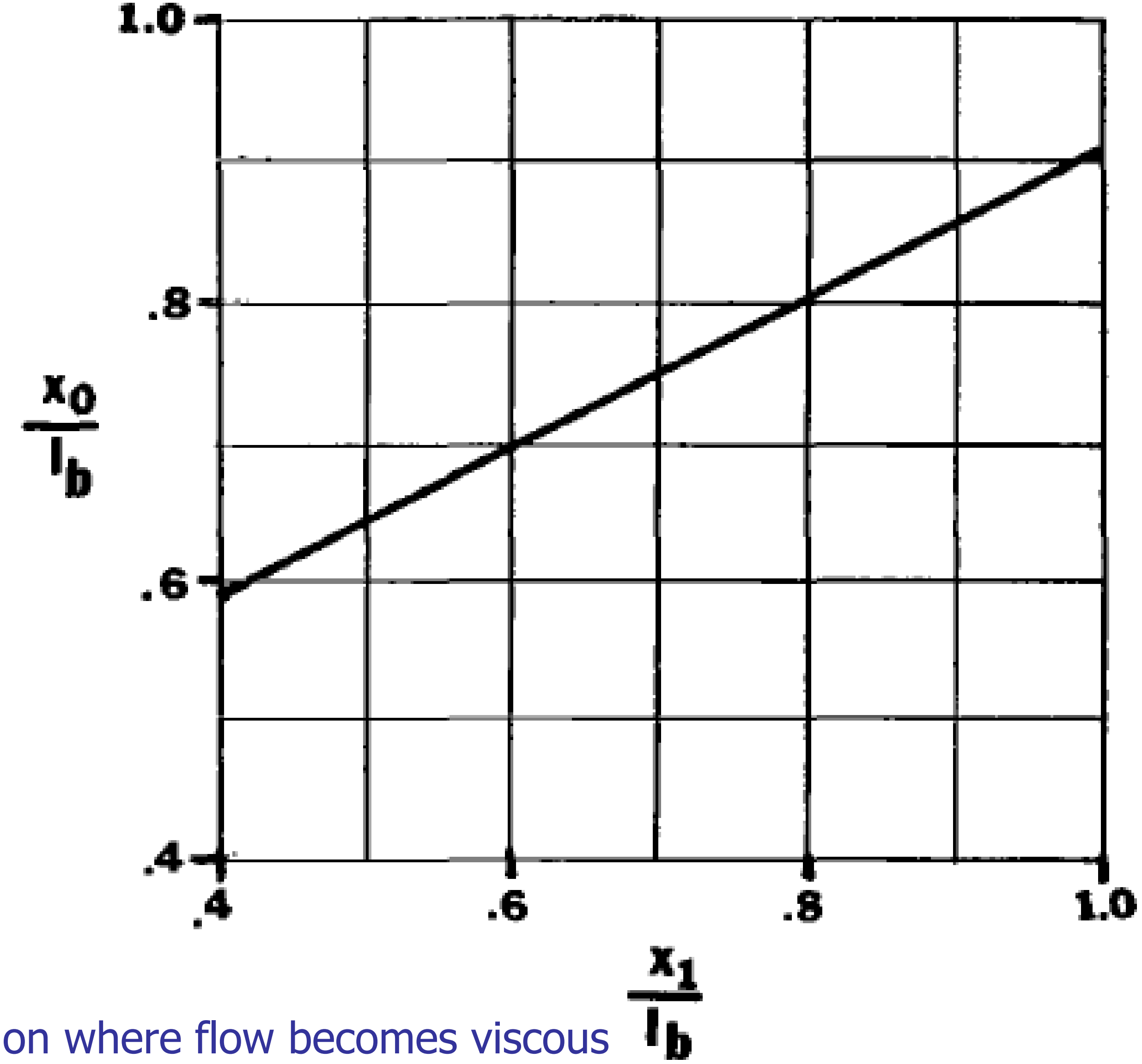


Reduced Mass Factor

Fig A6

x_0 = body station where flow ceases to be potential,

x_1 = the body station where the parameter dS_x/dx first reaches its minimum value.



Body station where flow becomes viscous

Wing-Fuselage Contribution $C_{L_{\alpha_{WB}}}$

Estimation of lift-curve slope. The lift-curve slope of the combined wingbody is given by

$$C_{L\alpha,WB} = \left[K_N + K_{W(B)} + K_{B(W)}\right] C_{L\alpha,e} \frac{S_{\text{exp}}}{S}$$

K_N → Ratio of nose lift ratio
$K_{W(B)}$ → Ratio of the wing lift in presence of the body
$K_{B(W)}$ → Ratio of body lift in presence of the wing to wing-alone lift

$$K_N = \left(\frac{C_{L\alpha,N}}{C_{L\alpha,e}}\right) \frac{S}{S_{\text{exp}}}$$

$C_{L_{\alpha,N}}$ → lift-curve slope of the isolated nose,
$C_{L_{\alpha,e}}$ → lift-curve slope of the exposed wing → 1ª aproximación $C_{L_{\alpha,e}} \approx C_{L_{\alpha_W}}$
S_{exp} → exposed wing area,
S → reference (wing) area.

$$C_{L_{\alpha_{Body}}} = \frac{2(k_2 - k_1)\, S_{B,max}}{S}$$

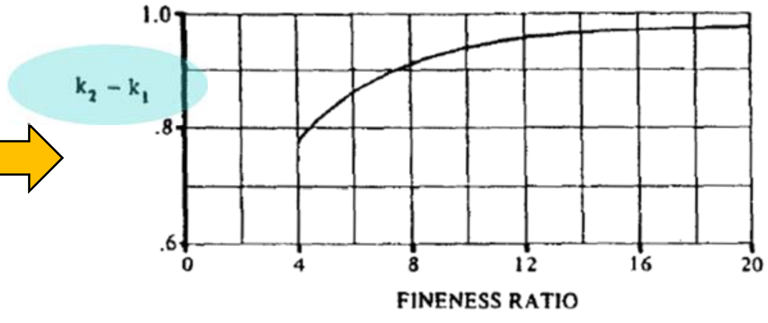
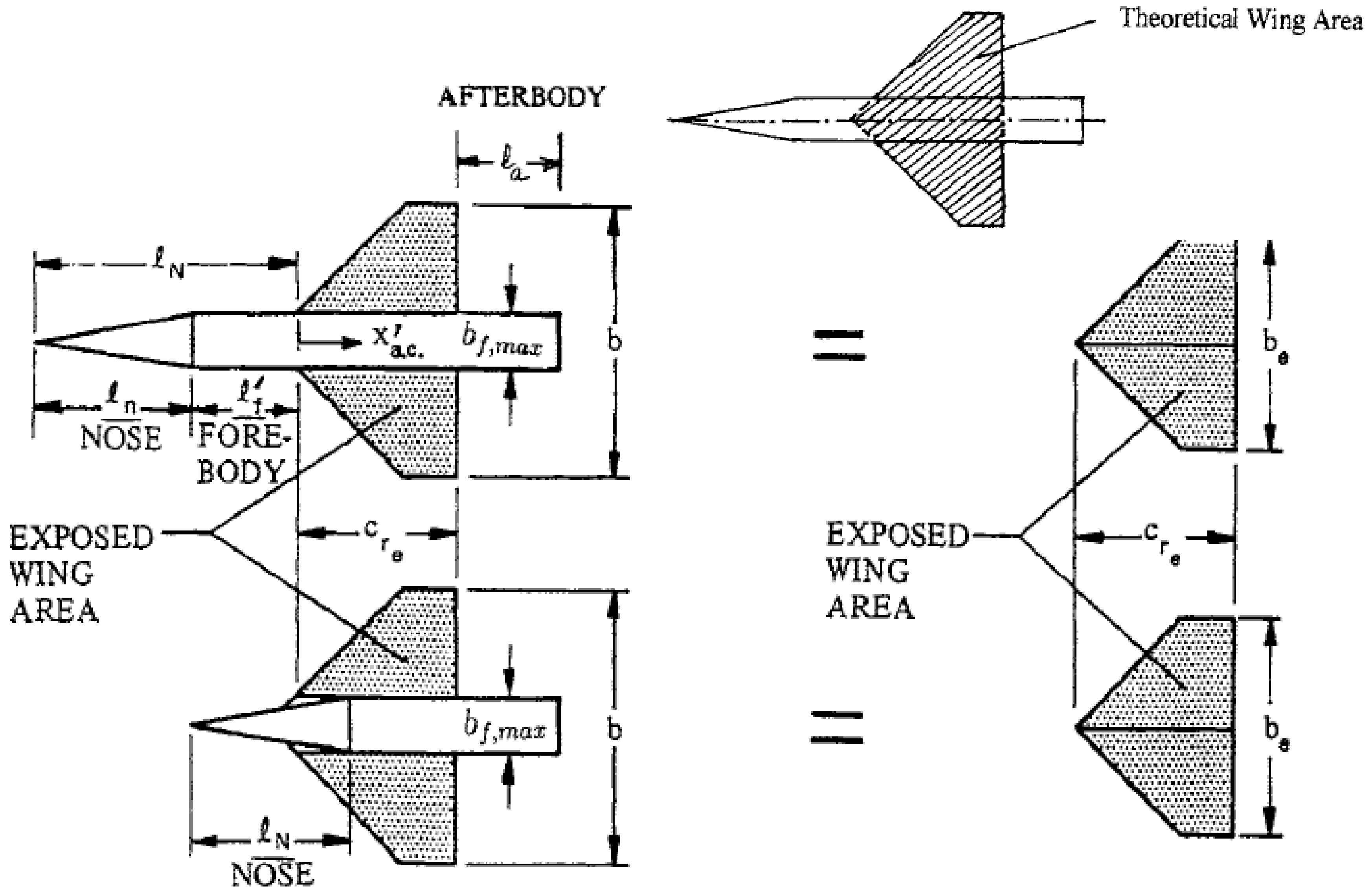


Fig. 3.6 Fuselage apparent mass coefficient.[1]

$k_2 - k_1$ is the apparent mass constant
$S_{B,max}$ is the maximum cross-sectional area of the fuselage

Wing-Fuselage Contribution $C_{L_{\alpha_{WB}}}$

Estimation of lift-curve slope. The lift-curve slope of the combined wingbody is given by



Wing-Fuselage Contribution $C_{L\alpha_{WB}}$

$$C_{L\alpha,WB} = \left[K_N + K_{W(B)} + K_{B(W)}\right] C_{L\alpha,e} \frac{S_{\text{exp}}}{S}$$

$$K_{W(B)} = 0.1714\left(\frac{b_{f,\max}}{b}\right)^2 + 0.8326\left(\frac{b_{f,\max}}{b}\right) + 0.9974$$

$$K_{B(W)} = 0.7810\left(\frac{b_{f,\max}}{b}\right)^2 + 1.1976\left(\frac{b_{f,\max}}{b}\right) + 0.0088$$

b_{max} → maximum width of the fuselage

b → wing span.

Fig A4

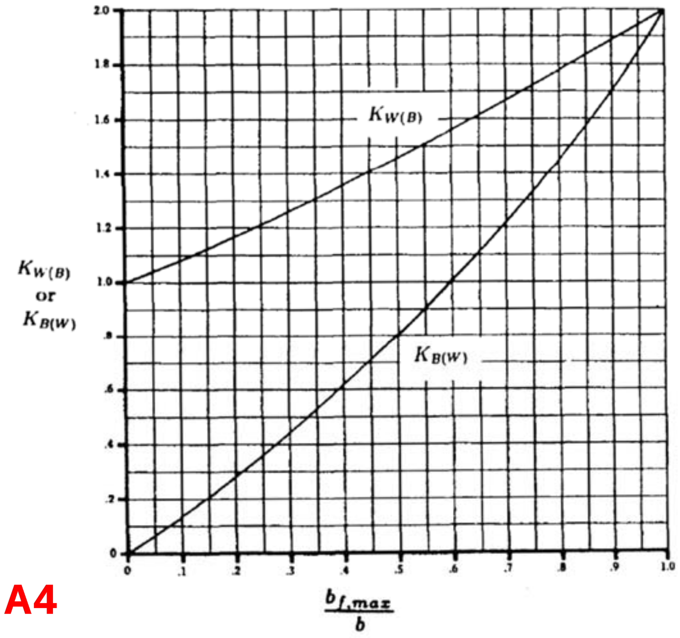
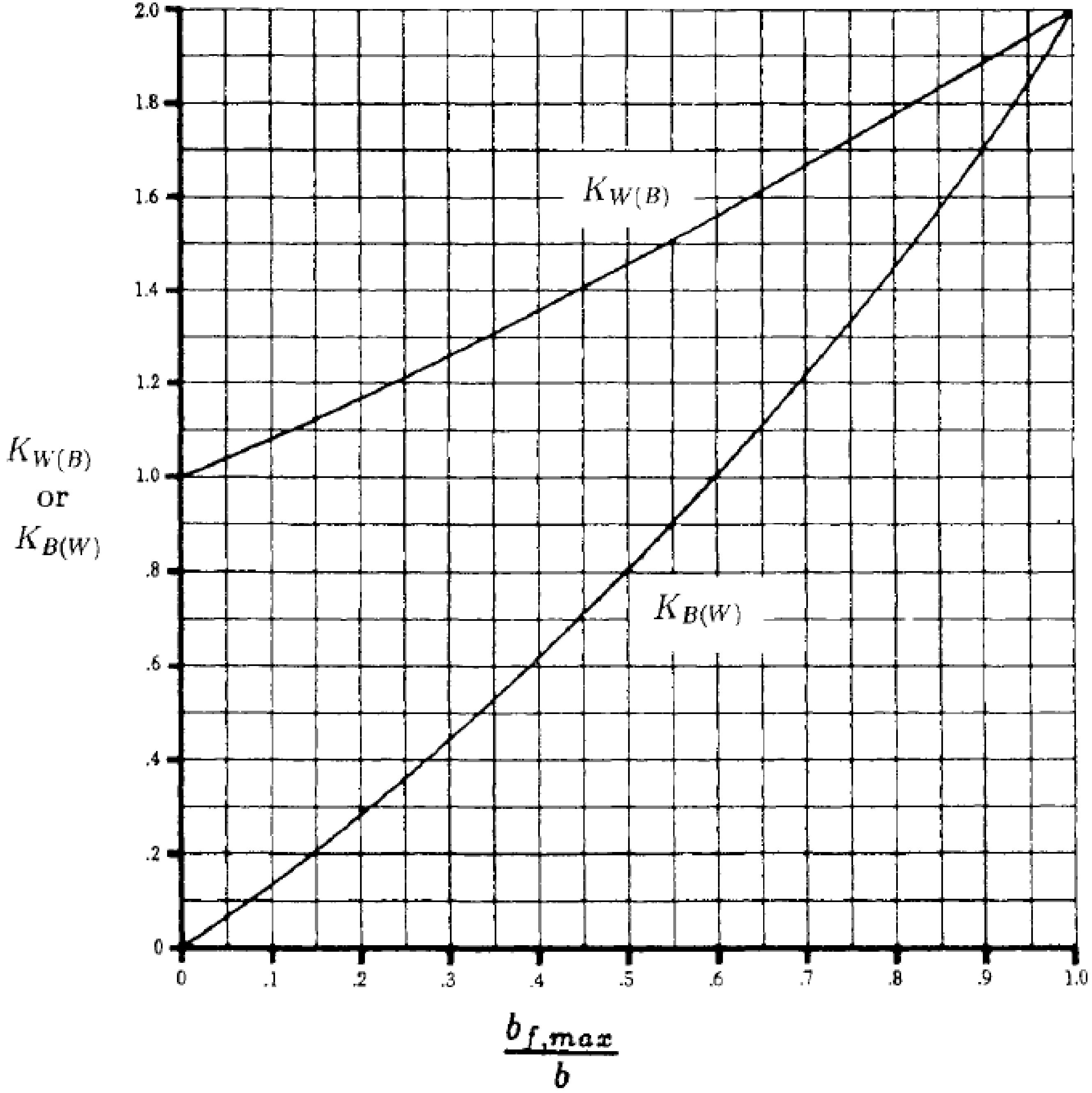


Fig. 3.17 Lift ratios $K_{B(W)}$ and $K_{W(B)}$ (Ref. 1).

Fig A4



2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
0
$K_{W(B)}$ or $K_{B(W)}$
$K_{W(B)}$
$K_{B(W)}$
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
$\frac{b_{f,max}}{b}$

C_{M_α} of the entire Airplane

$$\begin{aligned}\Sigma F_x &= W - L = \frac{W}{qS} - C_{L_0} - C_{L_\alpha}\alpha - C_{L_{\delta_e}}\delta_e \\ \Sigma M &= 0 = C_{M_0} + C_{M_\alpha}\alpha + C_{M_{\delta_e}}\delta_e\end{aligned}$$

$$\begin{aligned}C_{M_\delta} &= C_{M_{\delta_c}} + C_{M_{\delta_t}} \\ C_{M_{\delta_c}} &= \frac{q_c}{q}\frac{S_c}{S}(\bar{X}_{CG} - \bar{X}_{AC_c})C_{L_{\delta_c}} + \frac{q_c}{q}\frac{S_c}{S}\frac{\bar{C}_c}{\bar{C}}C_{M_{\delta_{AC_c}}} \\ C_{M_{\delta_t}} &= \frac{q_t}{q}\frac{S_t}{S}(\bar{X}_{CG} - \bar{X}_{AC_t})C_{L_{\delta_t}} + \frac{q_t}{q}\frac{S_t}{S}\frac{\bar{C}_t}{\bar{C}}C_{M_{\delta_{AC_h}}}\end{aligned}$$

$$\begin{aligned}C_{M_0} &= \frac{q_c}{q}\frac{S_c}{S}\frac{\bar{c}_c}{\bar{c}}C_{M_{AC_c}} + \frac{q_c}{q}\frac{S_c}{S}(\bar{X}_{CG} - \bar{X}_{AC_c})(C_{L_{0_c}} + C_{L_{\alpha_c}}(i_c + \epsilon_{0_c})) \\ &+ C_{M_{p_0}} + C_{M_{AC_w}} + (\bar{X}_{CG} - \bar{X}_{AC_w})(C_{L_{0_w}} + C_{L_{\alpha_w}} i_w) \\ &+ \frac{q_t}{q}\frac{S_t}{S}\frac{\bar{c}_t}{\bar{c}}C_{M_{AC_t}} + \frac{q_t}{q}\frac{S_t}{S}(\bar{X}_{CG} - \bar{X}_{AC_t})(C_{L_{0_t}} + C_{L_{\alpha_t}}(i_h + \epsilon_{0_t})) \\ C_{M_\alpha} &= C_{L_\alpha}(\bar{X}_{CG} - \bar{X}_{NA})\end{aligned}$$

Momento cabeceo
planta propulsora
Asumir inicialmente =0

$C_{M_{\alpha_{WB}}}$ → representa la pendiente de sustentación del conjunto ala-fuselaje

En 1ª hipótesis sólo las superficies aerodinámicas generan sustentación

En 2ª hipótesis se puede estimar la contribución del fuselaje $\left(C_{M_{0_{fus}}}\ y C_{M_{\alpha_{fus}}}\right)$

- Mediante métodos experimentales : análisis software XFLR5
- Mediante métodos empíricos: ecuaciones analíticas función de geometría

$C_{M_{0,f}}$

for cambered fuselages such as those with leading-edge droop or aft upsweep,

$$C_{mo,f} = \frac{k_2 - k_1}{36.5 S\bar{c}} \int_0^{l_f} b_f^2 (\alpha_{ow} + i_{cl,B}) \, \mathrm{d}x$$

$k_2 - k_1$ is the apparent mass constant

$a_{0,w}$ is the wing zero-lift angle relative to the fuselage reference line

$i_{cl,B}$ is the incidence angle of the fuselage camberline relative to the fuselage reference line.

The parameter $i_{cl,B}$ is assumed to be negative for nose droop or aft upsweep

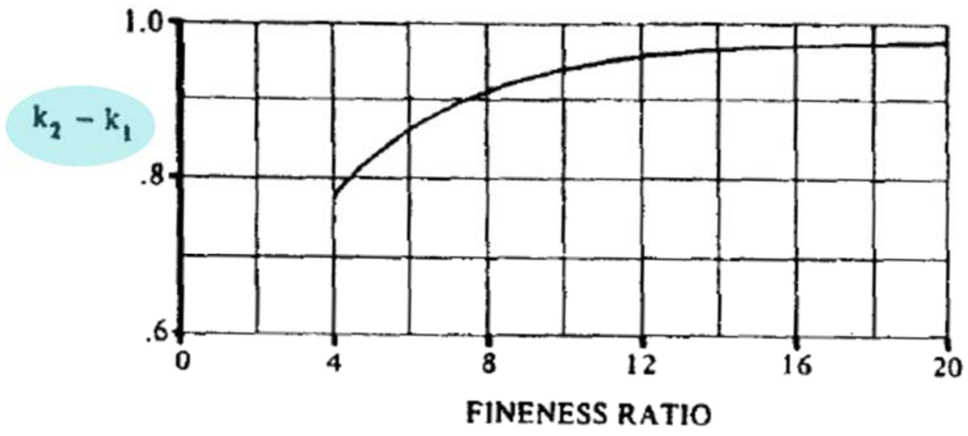


Fig A5 Fig. 3.6 Fuselage apparent mass coefficient.[1]

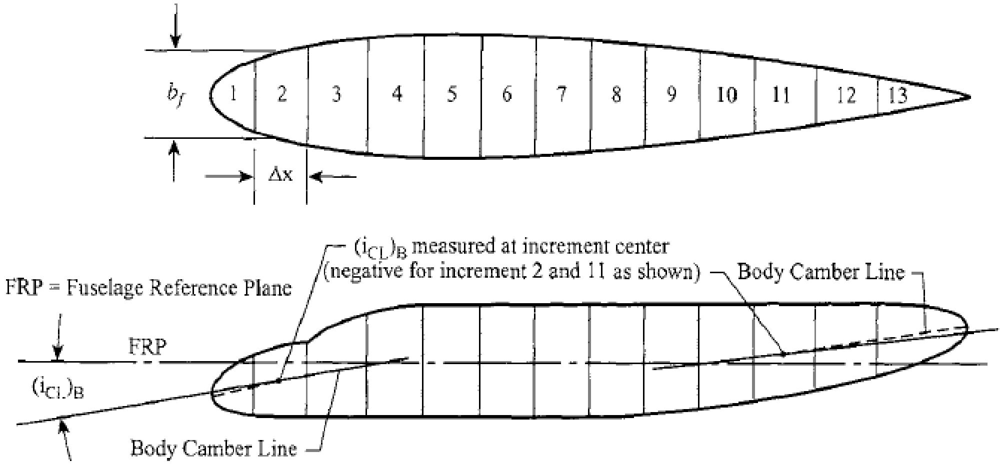


Fig A6 y A7

Fig A6

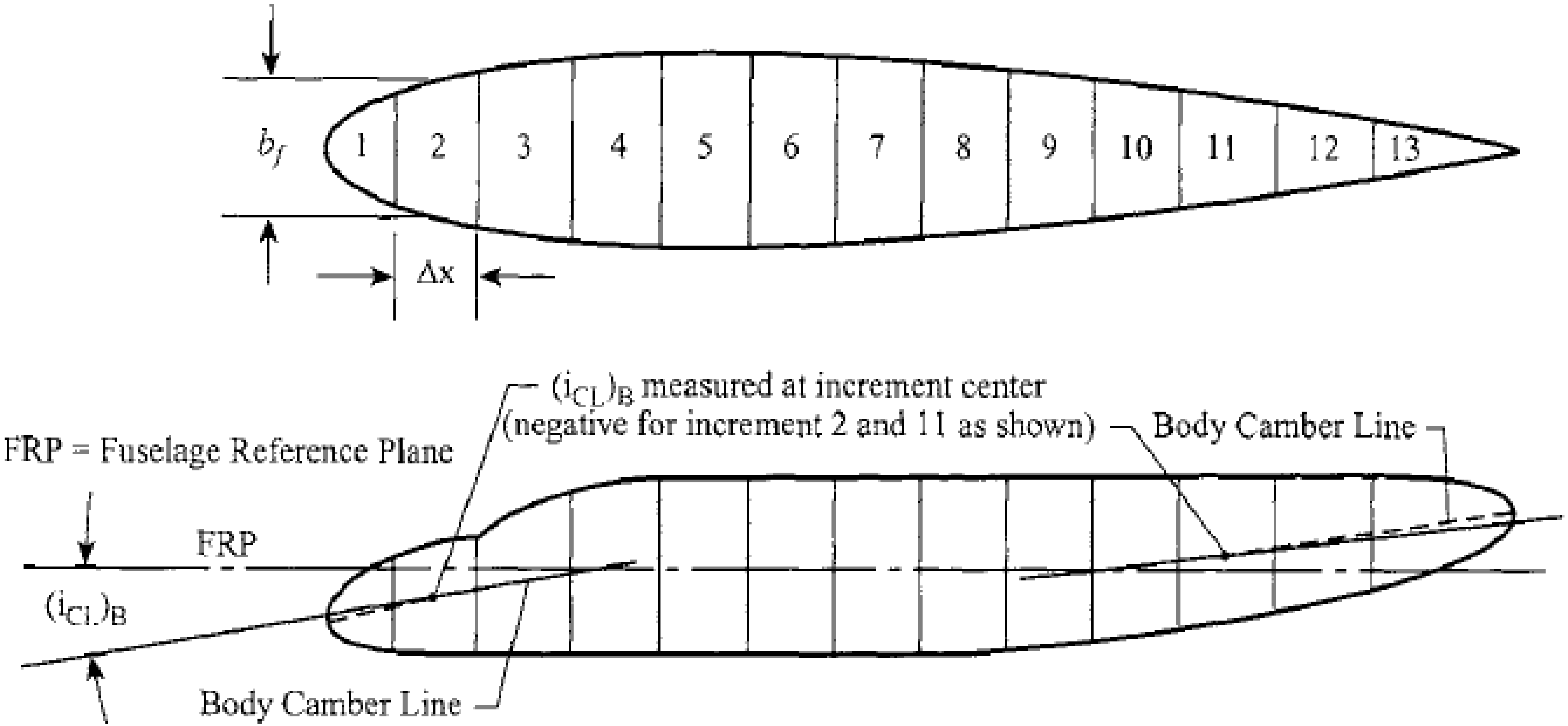


Fig. 3.8 Definition of fuselage nose droop and aft upsweep.[1]

Fig A7

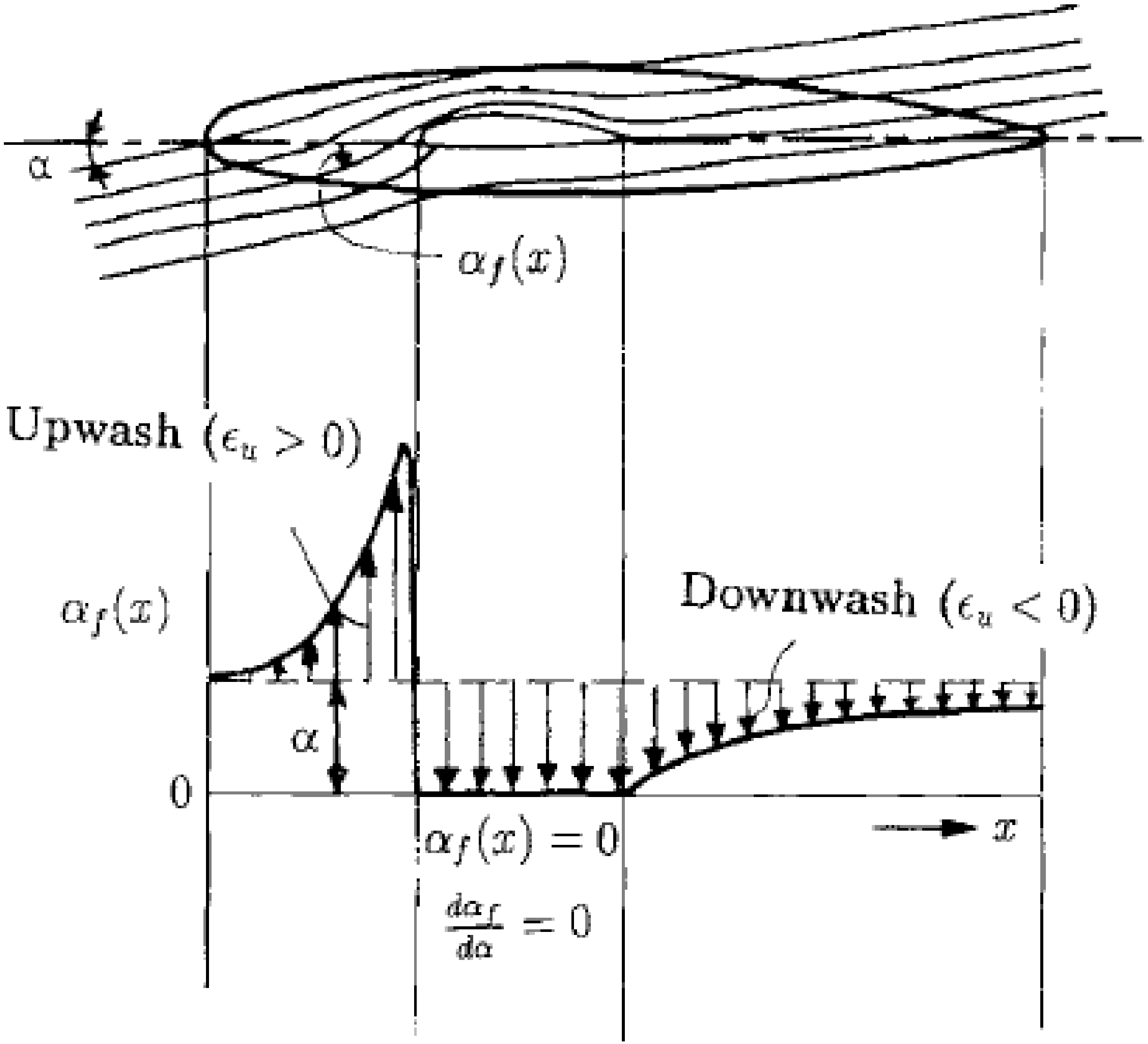


Fig. 3.7 Schematic diagram of the fuselage flow field in the presence of the wing.

$C_{M\alpha,f}$

Método 1

Estimación para fuselajes o nacelles

$$C_{M_a} = \frac{k_f w_f^2 l_b}{S_w c}$$

k_f = empirical factor Fig A8
w_f = maximum width of the fuselage or nacelle
l_b= length of fuselage or nacelle

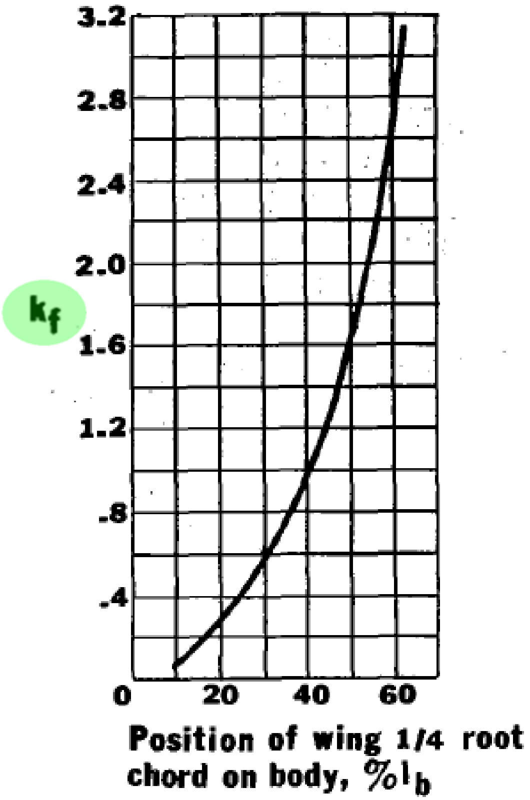
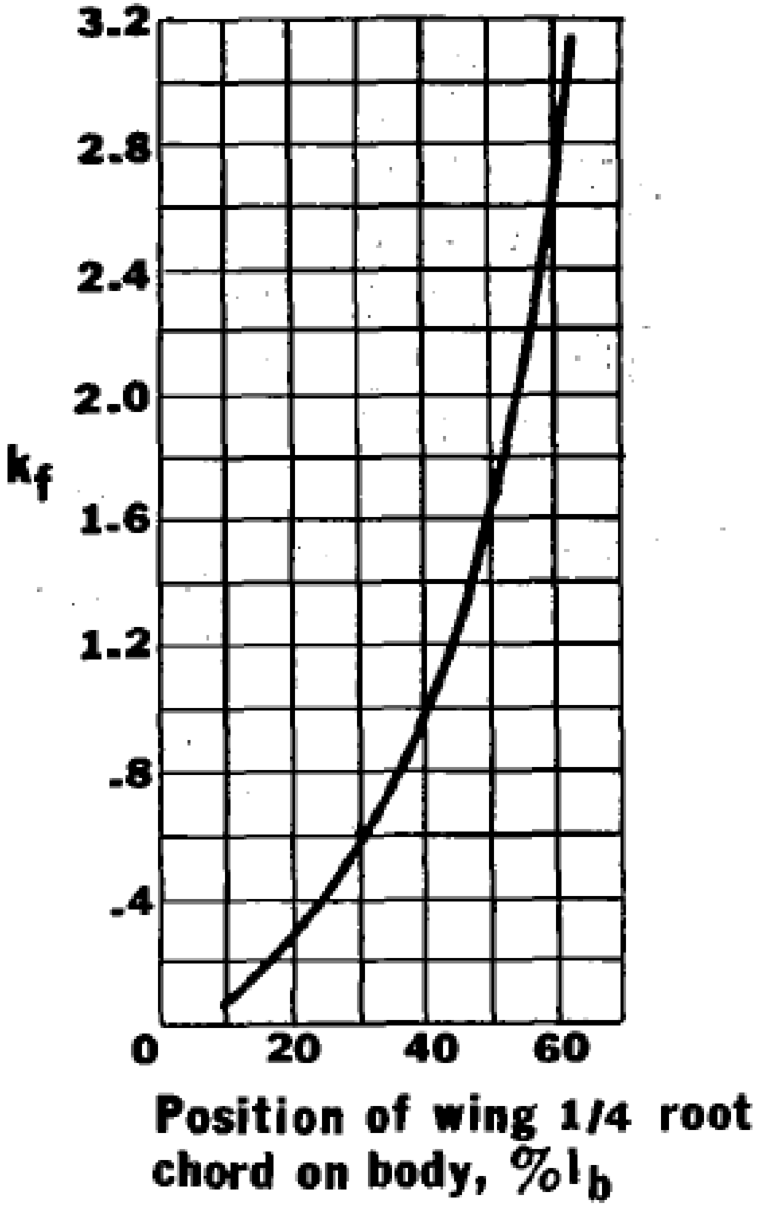


Fig A8

Fig A8



k_f = empirical factor for fuselage or nacelle contribution to $C_{M_{\alpha,f}}$

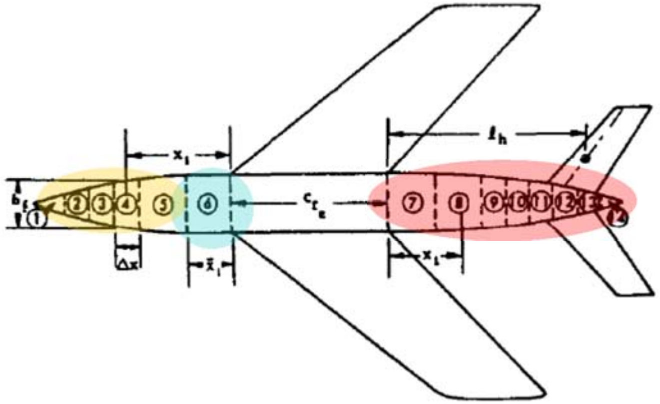
$C_{M_{\alpha,f}}$

Method 2: Multhopp modified Munk’s theory

$$\left(\frac{\partial C_m}{\partial \alpha}\right)_f = \frac{\pi}{2S\bar{c}} \int_0^{l_f} b_f^2 \left(1 + \frac{\partial \epsilon_u}{\partial \alpha}\right) \mathrm{d}x$$

Método más complejo

b_f → local width/diameter,
l_f → fuselage length,
S → reference (wing) area
c → lreference length (wing mean aerodynamic chord).



Se analiza la contribución del fuselaje en 3 zonas

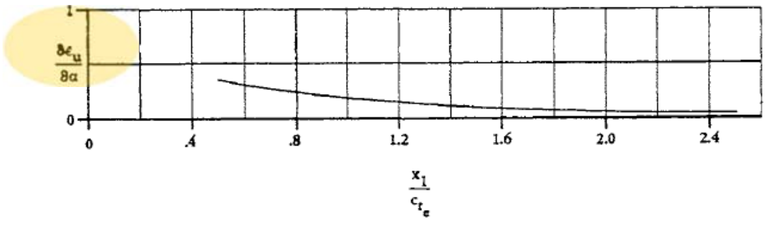
1. Zona alejada de la influencia up-wash (segmentos 1-5 ejemplo)
2. Zona bajo influencia up-wash (segmento 6 ejemplo)
3. Zona bajo influencia down-wash (segmentos 7-14 ejemplo)

$C_{M_{\alpha,f}}$

1. Zona alejada de la influencia up-wash (segmentos 1-5 ejemplo)

La distancia x_1 se mide desde el borde de ataque
Al punto medio de cada sección

$$\left(\frac{\partial C_m}{\partial \alpha}\right)_f = \frac{\pi}{2S\bar{c}} \int_0^{l_f} b_f^2 \left(1 + \frac{\partial \epsilon_u}{\partial \alpha}\right) \mathrm{d}x \Rightarrow \frac{\partial \epsilon_u}{\partial \alpha} \Rightarrow$$



2. Zona bajo influencia up-wash (segmento 6 ejemplo)

$$\left(\frac{\partial C_m}{\partial \alpha}\right)_f = \frac{\pi}{2S\bar{c}} \int_0^{l_f} b_f^2 \left(1 + \frac{\partial \epsilon_u}{\partial \alpha}\right) \mathrm{d}x \Rightarrow \frac{\partial \epsilon_u}{\partial \alpha} \Rightarrow$$

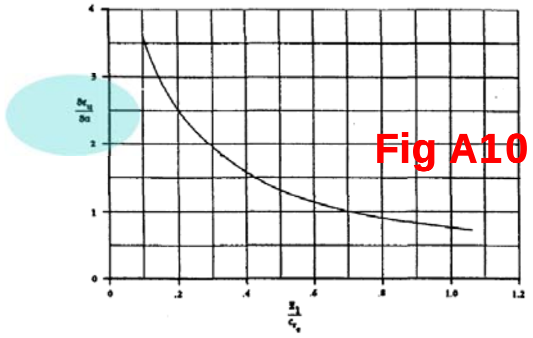
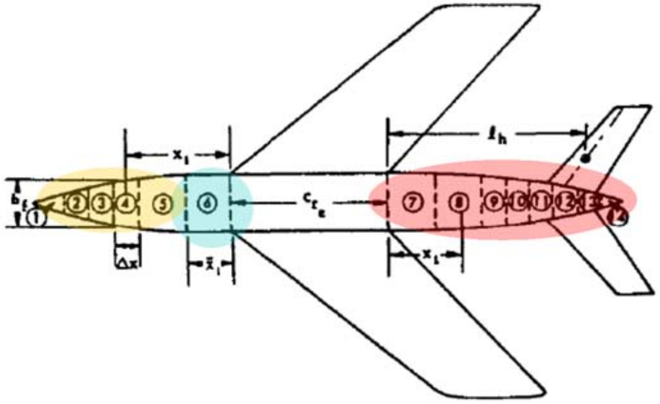
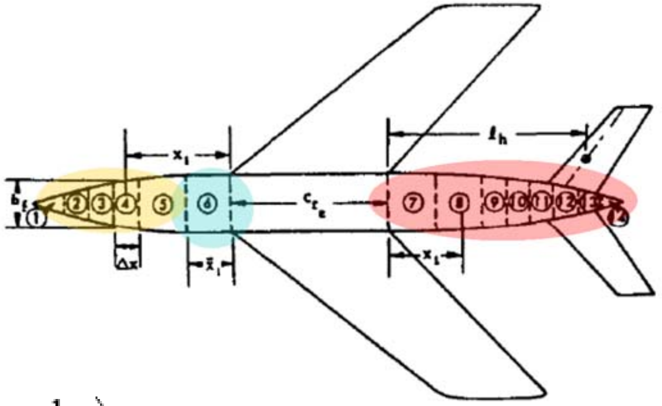


Fig A10



$C_{M\alpha,f}$



3 - Zona bajo influencia down-wash (segmentos 7-14 ejemplo)

$$\left(\frac{\partial C_m}{\partial \alpha}\right)_f = \frac{\pi}{2S\bar{c}} \int_0^{l_f} b_f^2 \left(1 + \frac{\partial \epsilon_u}{\partial \alpha}\right) \mathrm{d}x \Rightarrow 1 + \frac{\partial \epsilon_u}{\partial \alpha} = \frac{x_1}{l_h}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)$$

l_h is the distance (measured parallel to the root chord) between the trailing edge of the root chord and the horizontal tail aerodynamic center.

$$\frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha} = 4.44\left[K_A K_\lambda K_H (\cos \Lambda_{c/4})^{\frac{1}{2}}\right]^{1.19} \Rightarrow \tan \Lambda_{c/4} = \tan \Lambda_{LE} - \frac{C_r - C_t}{2b}$$

Here, K_A, K_λ and K_H are wing aspect ratio, wing taper ratio, and horizontal tail location factors

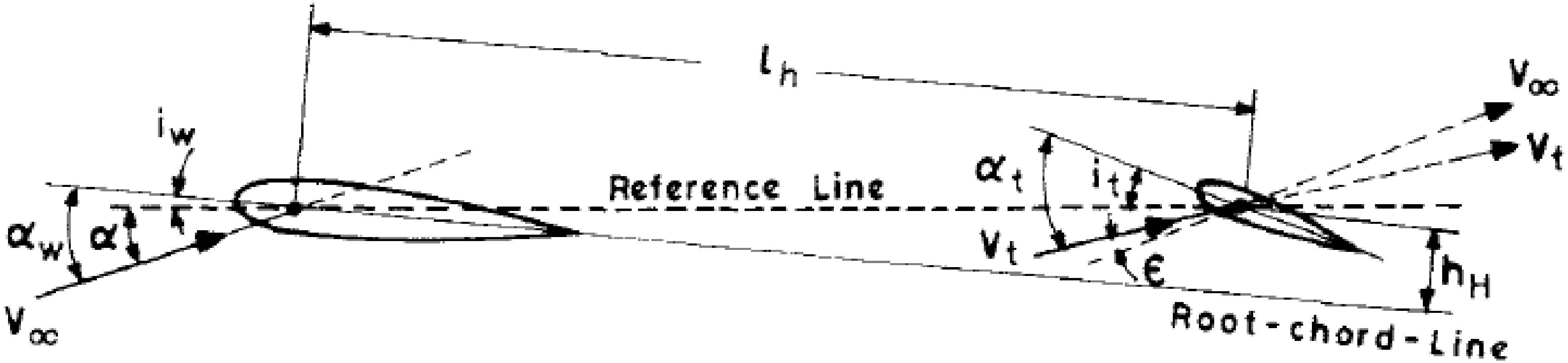
$$K_H = \frac{1 - \frac{h_H}{b}}{\sqrt[3]{\frac{2l_h}{b}}} \qquad K_A = \frac{1}{A} - \frac{1}{1 + A^{1.7}} \qquad K_\lambda = \frac{10 - 3\lambda}{7}$$

l_h →distance measured parallel to the wing root chord, between wing mac quarter chord point and the quarter chord point of the mac of horizontal tail

h_H → height of the horizontal tail mac above or below the plane of wing root chord, measured in the plane of symmetry and normal to the extended wing root chord and positive for horizontal tail mac above the plane of the wing root chord

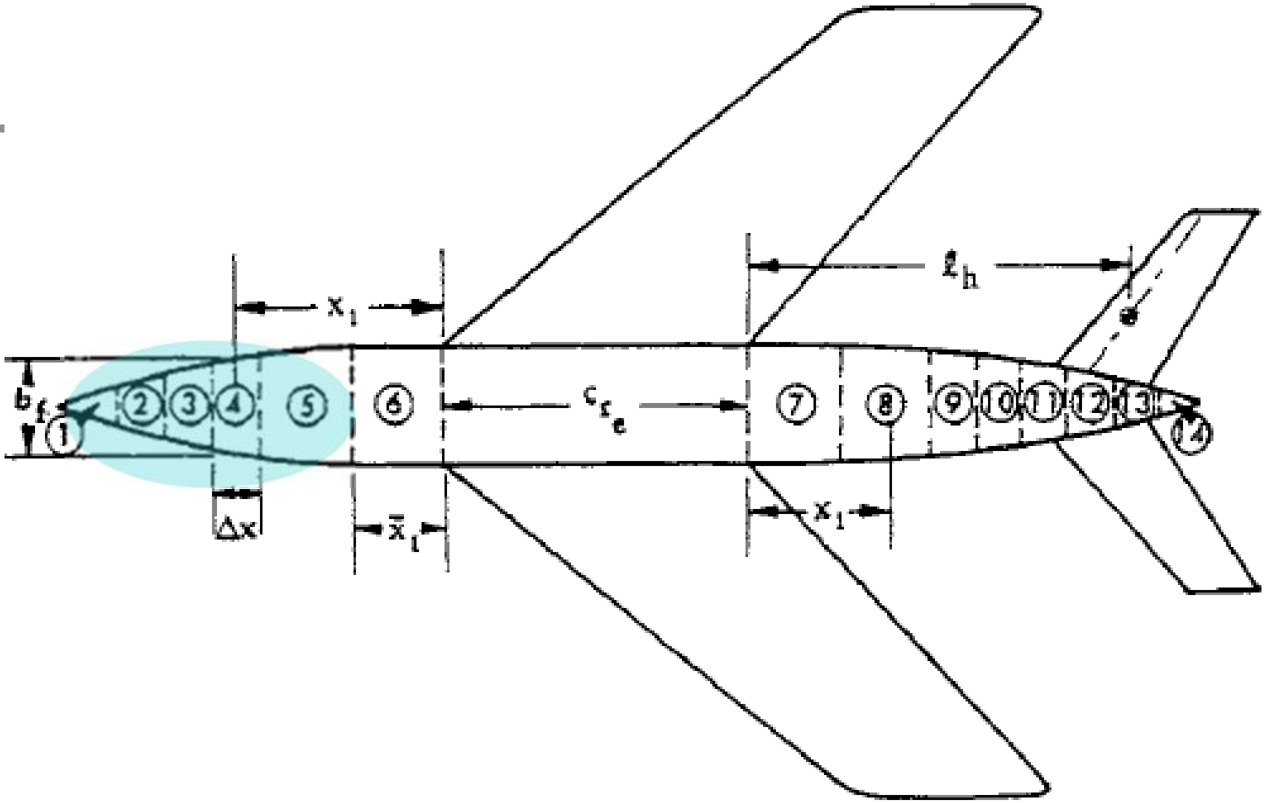
λ → taper ratio, A → Aspect Ratio

$C_{M_{\alpha,f}}$



b) Local flow directions at wing and tail

Fig A9



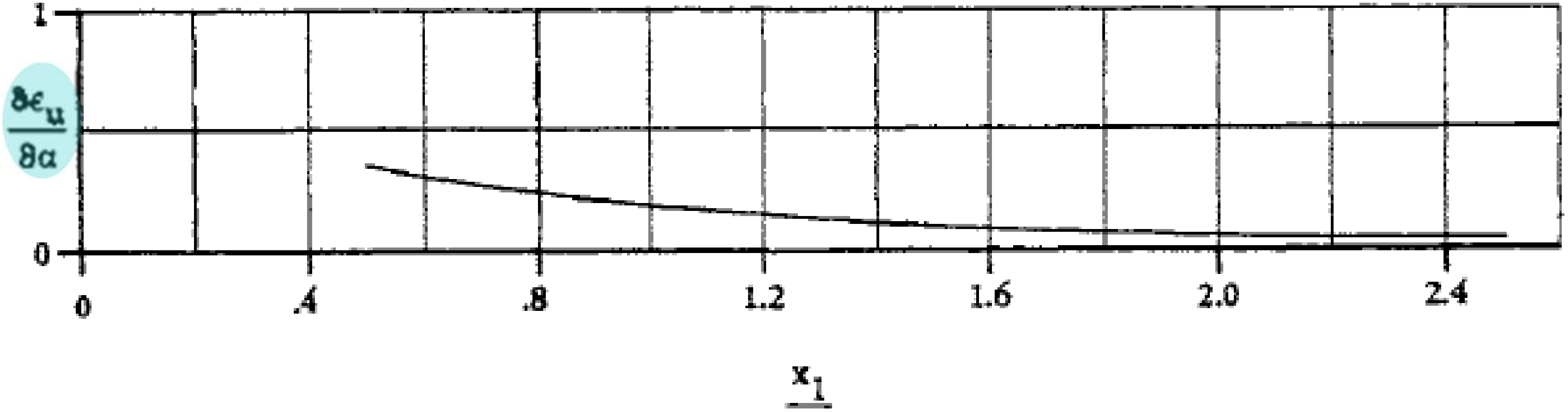


Fig. 3.9 Variation of fuselage upwash ahead of the wing.[1]

Fig A10

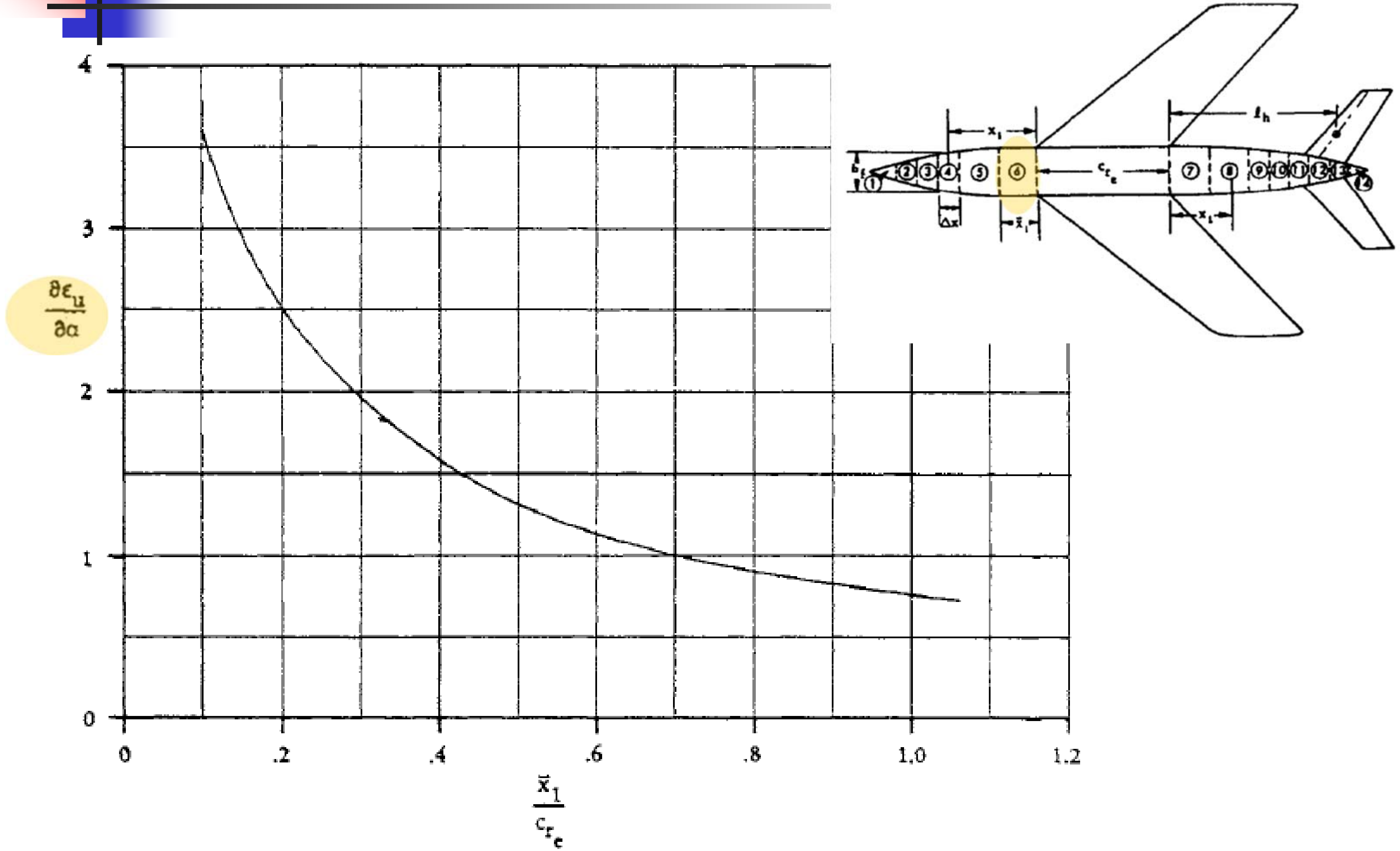


Fig. 3.9 Variation of fuselage upwash ahead of the wing.[1]

C_{M_α} of the Airplane

$$C_{M_{\alpha,airplane}} = C_{M_{\alpha_{canard}}} + C_{M_{\alpha_{tail}}} + C_{M_{\alpha_{wing}}} + C_{M_{\alpha_{fus/nacelles}}} + C_{M_{\alpha_{interference}}}$$

$$C_{M_{\alpha_{fus/nacelles}}} = \frac{k_f {w_f}^2 \, l_b}{S_w c}$$

k_f = empirical factor Fig A8
w_f = maximum width of the fuselage or nacelle
l_b= length of fuselage or nacelle

$$C_{M_{\alpha_{tail}}} = -\eta_t \frac{S_t}{S_w} \frac{l_t}{c} C_{L_{\alpha_t}} \left(1 - \frac{d\epsilon}{d\alpha}\right)$$

$$C_{M_{\alpha_{canard}}} = \eta_c \frac{S_c}{S_w} \frac{l_c}{c} C_{L_{\alpha_c}} \left(1 + \frac{d\epsilon_u}{d\alpha}\right)$$

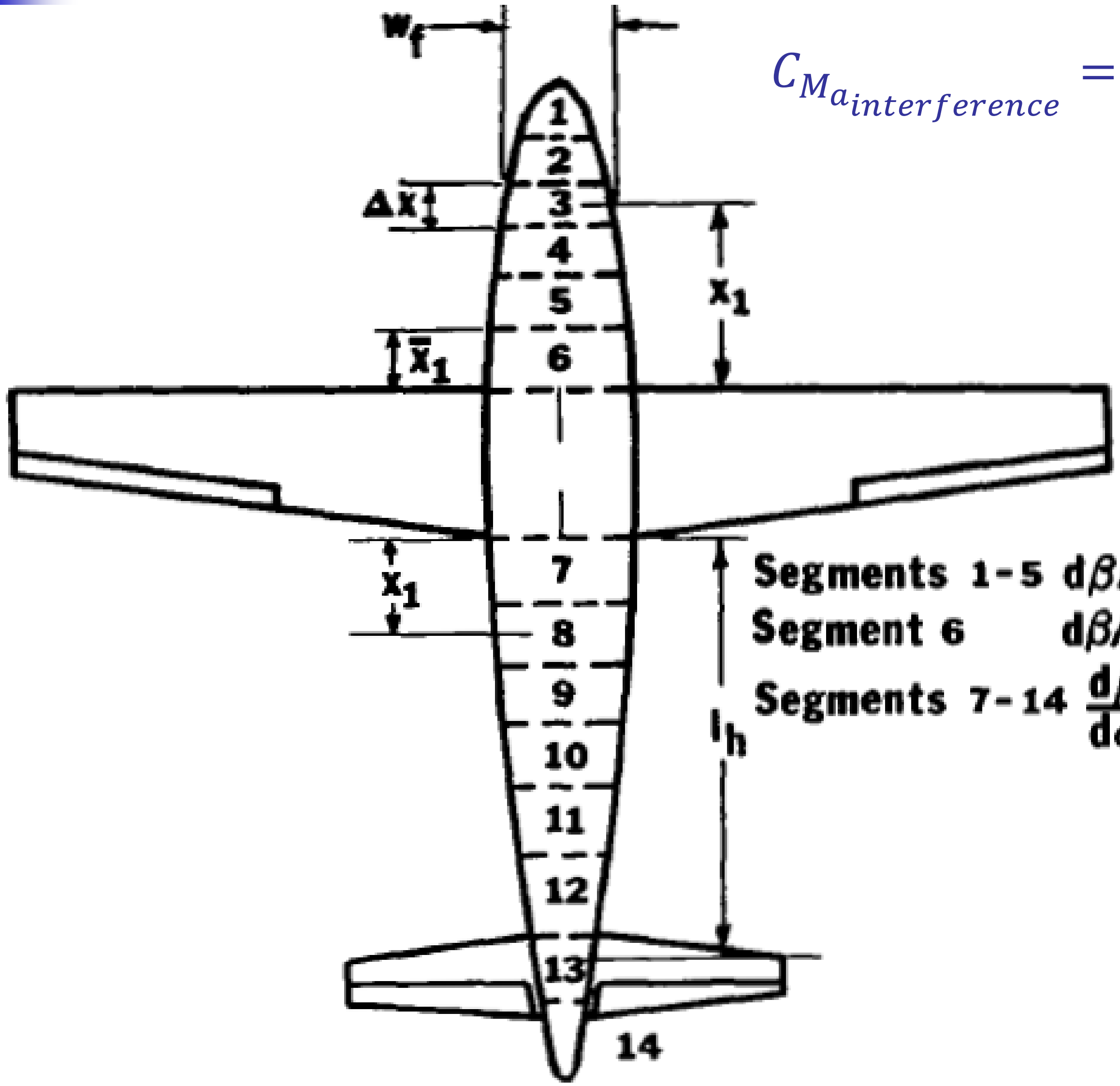
$$C_{M_{\alpha_{wing}}} = \left[\left[1 + \frac{2C_L}{\pi e AR}(\alpha - i_w) + \frac{C_D}{C_{L_\alpha}}\right]\frac{x_a}{c} + \left[\frac{2C_L}{\pi e AR} - (\alpha - i_w) - \frac{C_L}{C_{L_\alpha}}\right]\frac{z_a}{c}\right] C_{L_\alpha}$$

$$C_{M_{\alpha_{interference}}} = \frac{c}{290\,S}\left(w_{LE} + 2w_{Mid} - 3w_{TE}\right)$$

w_{LE}, w_{Mid}, w_{TE} width of the fuselage at the wing leading esge, mid chord, and trailing edge

C_{M_α} of the Airplane

$$C_{M_{\alpha_{interference}}} = \frac{c}{290\ S}\left(w_{LE} + 2w_{Mid} - 3w_{TE}\right)$$



Segments 1-5 $d\beta/d\alpha$ from curve a

Segment 6 $d\beta/d\alpha$ from curve b

Segments 7-14 $\frac{d\beta}{d\alpha} = \frac{x_1}{l_h}\left(1 - \frac{d\epsilon}{d\alpha}\right)$

$$\frac{dC_m}{d\alpha} = \frac{57.3}{36.5\ Sc}\sum_{s=1}^{s=14} w_f{}^2 \frac{d\beta}{d\alpha}\Delta x$$

Derivadas $C_{D_u}, C_{L_u}, C_{M_u}$

Speed Derivatives

Estimación Derivadas

- Contribución C_{D_u}

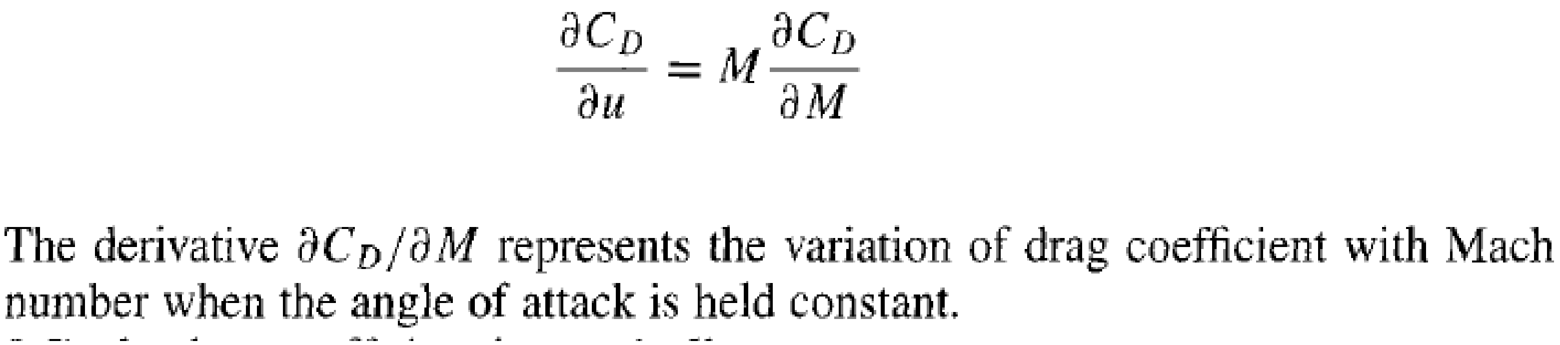
- Contribución C_{L_u}

- Contribución C_{M_u}

Derivadas en 1/rad si no se indica lo contrario
Si las derivadas no están en 1/rad hay que convertirlas

C_{D_u}

Estimación C_{D_u}



$$\frac{\partial C_D}{\partial u} = M\frac{\partial C_D}{\partial M}$$

The derivative $\partial C_D/\partial M$ represents the variation of drag coefficient with Mach number when the angle of attack is held constant.

At low subsonic speeds *(M* <0.5), the drag coefficient is practically constant ⇒ $\partial C_D/\partial M = 0.$

$$C_{D_u} \approx 0$$

As the flight Mach number approaches the critical Mach number M_{cr} the drag coefficient starts rising

It assumes a peak value in the transonic Mach number range and starts decreasing as Mach number becomes supersonic.

It tends to assume a steady value at high supersonic or hypersonic Mach numbers.

Therefore, if the flight Mach number exceeds 0.5, the derivative C_{D_u} should not be ignored

C_{L_u}

Estimación C_{D_u}

$$\frac{\partial C_L}{\partial u} = M\alpha \frac{\partial C_{L\alpha}}{\partial M}$$

At low subsonic speeds ($M \leq 0.5$), the lift-curve slope $C_{L\alpha}$ essentially remains constant so that $\partial C_{L\alpha}/\partial M = 0$.

At low subsonic speeds *(M* <0.5), the lift curve slope is practically constant $\Rightarrow$ $\partial C_{L\alpha}/\partial M = 0.$

$$C_{L_u} \approx 0$$

For C_L of the form

$$C_L = \frac{C_{L_0} + (C_{L_\alpha}|_{M=0})\alpha}{\sqrt{(1 - M^2)}}$$

then

$$C_{L_u} = \frac{\partial C_L}{\partial(\frac{u}{U_1})} = \frac{U_1}{a}\frac{\partial C_L}{\partial \frac{u}{a}} = M_1 \frac{\partial C_L}{\partial M} \Rightarrow C_{L_u} = \frac{M_1^2}{(1 - M_1^2)} C_L$$

C_{M_u}

Estimación C_{M_u}

$$\frac{\partial C_m}{\partial u} = M\alpha \frac{\partial C_{m\alpha}}{\partial M}$$

At low subsonic speeds *(M* <0.5), the drag coefficient is practically constant

Or equivalenty

$$C_{m_u} = M_1 \frac{\partial C_m}{\partial M}$$

with

$$\frac{\partial C_m}{\partial M}(\Delta M) = -\Delta \bar{x}_{ac_A} C_{L_1}$$

$\Delta \bar{x}_{ac_A}$ is the aft shift in airplane aerodynamic center for a change in Mach number,,

therefore

$$C_{m_u} = -M_1 C_{L_1} \frac{\partial x_{ac_A}}{\partial M}$$

$\frac{\partial x_{ac_A}}{\partial M}$ →variación del centro aerodinámico con cambio de Mach

$\frac{\partial x_{ac_A}}{\partial M} \approx 0$ para $M < 0.5$, hay que tenerla en cuenta para $M > 0.5$

Derivadas $C_{D_q}, C_{L_q}, C_{M_q}$

Pitch Rate Derivatives

Estimación Derivadas

- Contribución C_{D_q}

- Contribución C_{L_q}
  - Wing
  - Horizontal/V-tail/canard

- Contribución C_{M_q}
  - Wing
  - Horizontal/V-tail/canard

Derivadas en 1/rad si no se indica lo contrario
Si las derivadas no están en 1/rad hay que convertirlas

Pitch Rate Derivatives C_{D_q}

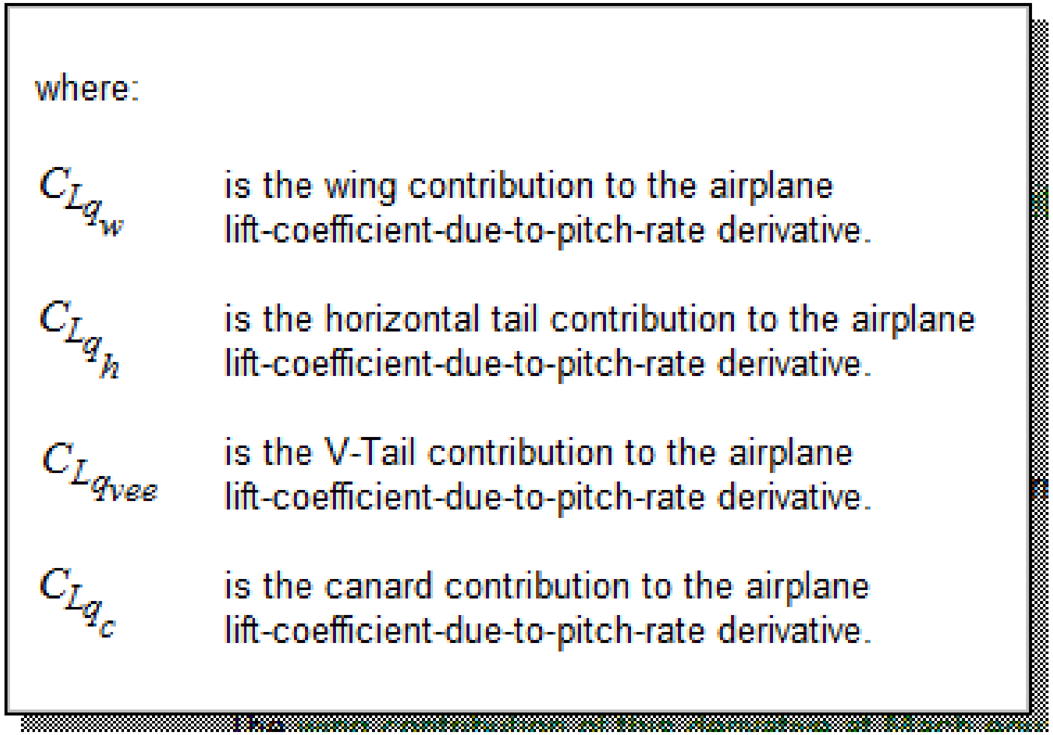
The airplane drag-coefficient-due-to-pitch-rate derivative is negligible

$$C_{D_q} \approx 0$$

Pitch Rate Derivatives C_{L_q}

The airplane lift-coefficient-due-to-pitch-rate derivative is

$$C_{L_q} = \overbrace{C_{L_{q_w}}}^{\text{wing}} + \underbrace{C_{L_{q_h}}}_{\text{horizontal}} + \overbrace{C_{L_{q_{vee}}}}^{\text{V-tail}} + \underbrace{C_{L_{q_c}}}_{\text{canard}}$$



where:

$C_{L_{q_w}}$	is the wing contribution to the airplane lift-coefficient-due-to-pitch-rate derivative.
$C_{L_{q_h}}$	is the horizontal tail contribution to the airplane lift-coefficient-due-to-pitch-rate derivative.
$C_{L_{q_{vee}}}$	is the V-Tail contribution to the airplane lift-coefficient-due-to-pitch-rate derivative.
$C_{L_{q_c}}$	is the canard contribution to the airplane lift-coefficient-due-to-pitch-rate derivative.

Contribución Ala C_{L_q}

Wing contribution

Método 1

$$C_{L_{q_W}} = \frac{AR_W + 2\cos\Lambda_{c/4_W}}{AR_W B + 2\cos\Lambda_{c/4_W}} C_{L_{q_W @ M=0}}$$

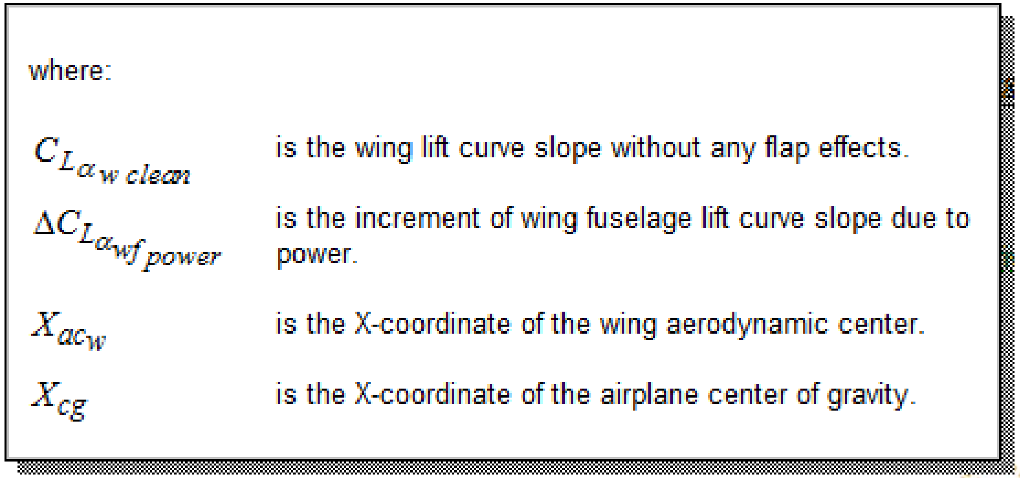
AR_w → wing aspect ratio
B → compressibility sweep correction factor
$\Lambda_{c/4_w}$ → is the wing quarter chord angle.

$C_{L_{q_W @ M=0}}$ → wing contribution to airplane lift-coefficient-due-to-pitch-rate derivative at Mach equals zero

$$B = \sqrt{1 - M_1{}^2 \cos\left(\Lambda_{c/4_W}\right)^2}$$

$$C_{L_{q_W @ M=0}} = \left(C_{L_{\alpha_W\,clean}} + \Delta C_{L_{\alpha_{Wf\,power}}}\right)\left[\frac{1}{2} + \frac{2(X_{ac_W} - X_{cg})}{\bar{c}_W}\right]$$

1ª Aproximación → $\Delta C_{L_{\alpha_{wf\,power}}} \approx 0$



where:

$C_{L_{\alpha_W\,clean}}$	is the wing lift curve slope without any flap effects.
$\Delta C_{L_{\alpha_{Wf\,power}}}$	is the increment of wing fuselage lift curve slope due to power.
X_{ac_W}	is the X-coordinate of the wing aerodynamic center.
X_{cg}	is the X-coordinate of the airplane center of gravity.

Contribución Ala C_{L_q}

The contribution of the wing-body combination

Método 2

$$(C_{Lq})_{WB} = [K_{W(B)} + K_{B(W)}]\left(\frac{S_e\bar{c}_e}{S\bar{c}}\right)(C_{Lq})_e + (C_{Lq})_B\left(\frac{S_{B,\max}l_f}{S\bar{c}}\right)$$

c_e mean aerodynamic chords of the exposed wing

c mean aerodynamic chords of the total (theoretical) wing

$\left(C_{L_q}\right)_e$ and $\left(C_{L_q}\right)_B$ → contributions of the exposed wing and isolated body

Velocidades subsónicas → $(C_{Lq})_e = \left(\frac{1}{2} + 2\xi\right)(C_{L\alpha})_e$ →

$$\xi = \frac{\bar{x}}{\bar{c}_e}$$

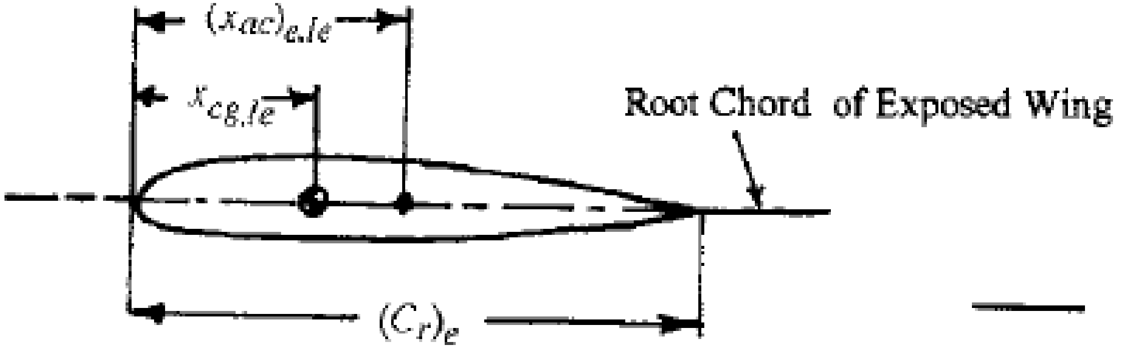
$$\bar{x} = (x_{ac})_e - x_{cg,le}$$

$(x_{ac})_e$ → distance of exposed wing aerodynamic center from the leading edge of the root chord

$x_{cg\,le}$ → distance of the center of gravity from the leading edge of the exposed wing root chord.

$(x_{ac})_e$ and $x_{cg\,le}$ are measured parallel to the exposed wing root chord.

The parameter $\bar{x}$ will be positive if the aerodynamic center of the exposed wing $(x_{ac})_e$ is aft of the center of gravity



Wing-Fuselage Contribution C_{L_q}

Método 2

$$(C_{Lq})_{WB} = [K_{W(B)} + K_{B(W)}]\left(\frac{S_e\bar{c}_e}{S\bar{c}}\right)(C_{Lq})_e + (C_{Lq})_B\left(\frac{S_{B,\max}l_f}{S\bar{c}}\right)$$

$$K_{W(B)} = 0.1714\left(\frac{b_{f,\max}}{b}\right)^2 + 0.8326\left(\frac{b_{f,\max}}{b}\right) + 0.9974$$

$$K_{B(W)} = 0.7810\left(\frac{b_{f,\max}}{b}\right)^2 + 1.1976\left(\frac{b_{f,\max}}{b}\right) + 0.0088$$

b_{max} → maximum width of the fuselage
b → wing span.

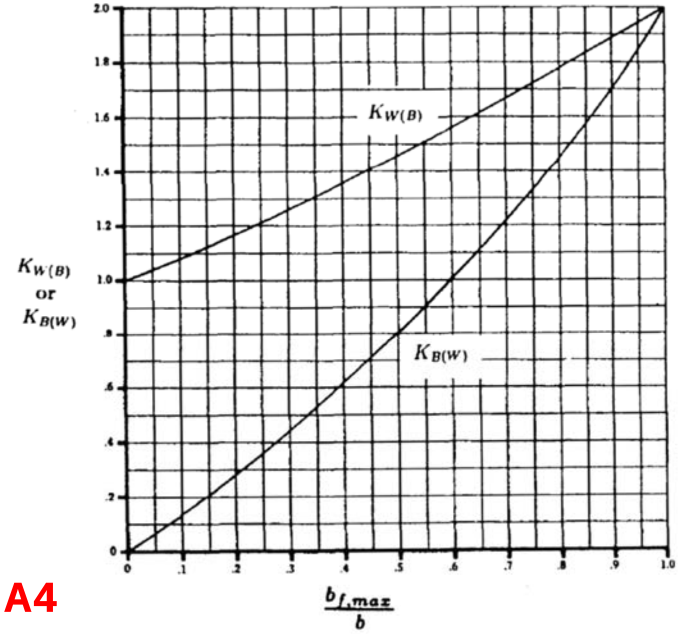
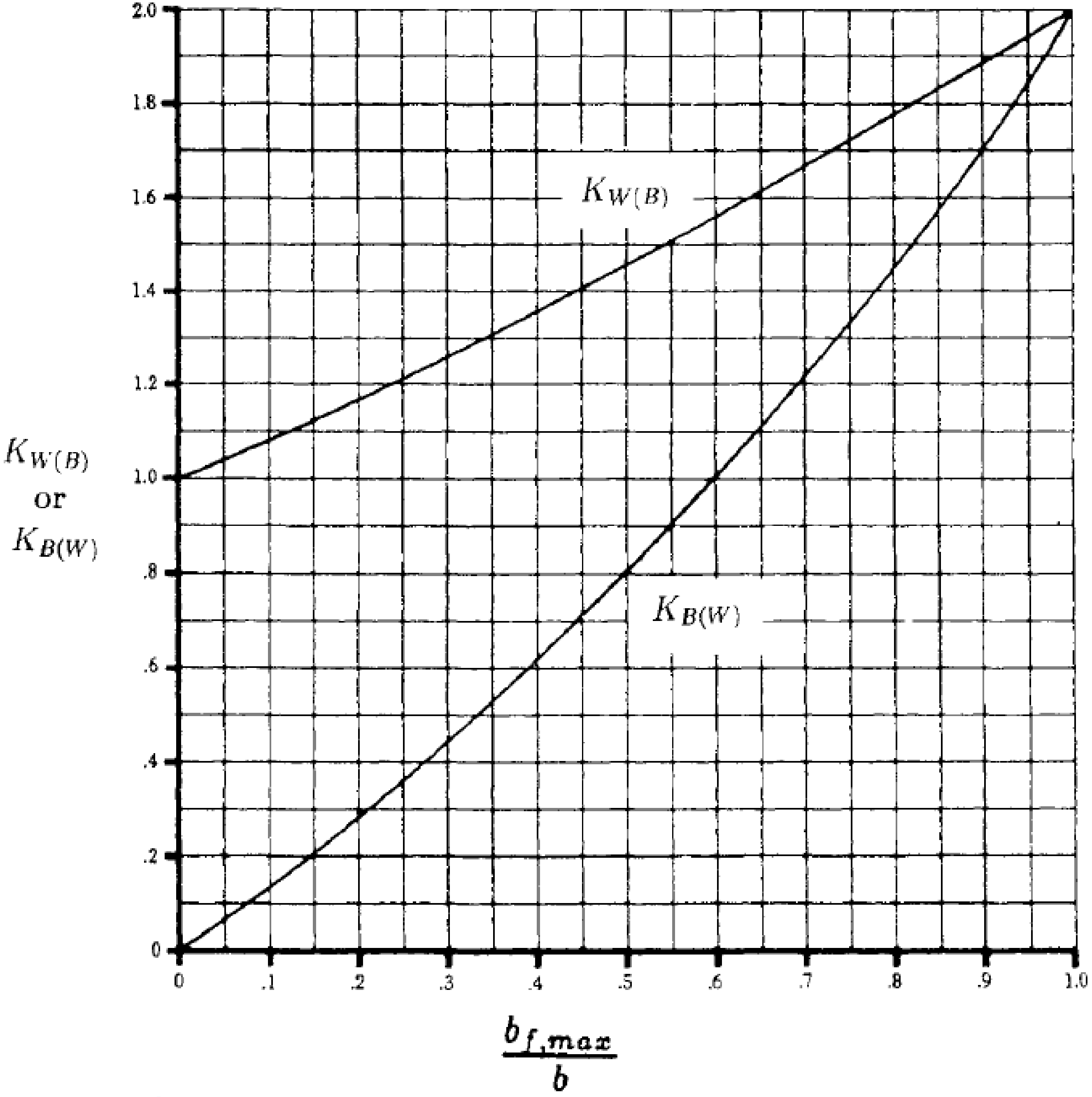


Fig A4

Fig. 3.17 Lift ratios $K_{B(W)}$ and $K_{W(B)}$ (Ref. 1).

Fig A4

Método 2



Contribución Ala C_{L_q}

The contribution of the wing-body combination

Método 2

$$(C_{Lq})_{WB} = [K_{W(B)} + K_{B(W)}]\left(\frac{S_e\bar{c}_e}{S\bar{c}}\right)(C_{Lq})_e + (C_{Lq})_B\left(\frac{S_{B,\max}l_f}{S\bar{c}}\right)$$

c_e mean aerodynamic chords of the exposed wing

c mean aerodynamic chords of the total (theoretical) wing

$\left(C_{L_q}\right)_e$ and $\left(C_{L_q}\right)_B$ → contributions of the exposed wing and isolated body

Velocidades subsónicas → $(C_{Lq})_e = \left(\frac{1}{2} + 2\xi\right)(C_{L\alpha})_e$ → $\xi = \frac{\bar{x}}{\bar{c}_e}$

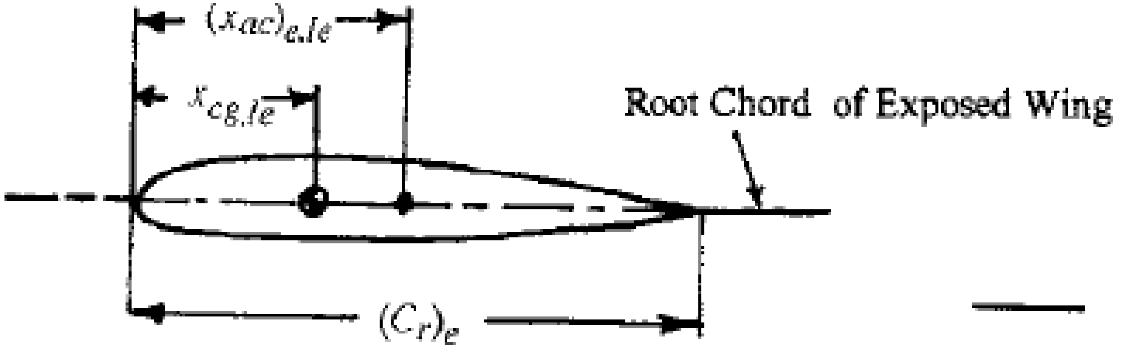
$$\bar{x} = (x_{ac})_e - x_{cg,le}$$

$(x_{ac})_e$ → distance of exposed wing aerodynamic center from the leading edge of the root chord

$x_{cg\,le}$ → distance of the center of gravity from the leading edge of the exposed wing root chord.

$(x_{ac})_e$ and $x_{cg\,le}$ are measured parallel to the exposed wing root chord.

The parameter $\bar{x}$ will be positive if the aerodynamic center of the exposed wing $(x_{ac})_e$ is aft of the center of gravity



Contribución Ala C_{L_q}

The body contribution

Método 2

$$(C_{Lq})_{WB} = [K_{W(B)} + K_{B(W)}]\left(\frac{S_e\bar{c}_e}{S\bar{c}}\right)(C_{Lq})_e + (C_{Lq})_B\left(\frac{S_{B,\max}l_f}{S\bar{c}}\right)$$

$$(C_{Lq})_B = 2\left(C'_{L\alpha}\right)_B\left(1-\frac{x_m}{l_f}\right) \Rightarrow$$

$$\left(C'_{L\alpha}\right)_B = (C_{L\alpha})_B\left(\frac{V_B^{2/3}}{S_{B,\max}}\right)$$

$$(C_{L\alpha})_B = 2(k_2-k_1)\left(\frac{S_{B,\max}}{V_B^{2/3}}\right)$$

$k_2 - k_1$ is the apparent mass constant

$S_{B,max}$ is the maximum cross-sectional area of the fuselage,

l_f total length of the fuselage

V_B volume of the fuselage.

$$\left(C_{L_q}\right)_B = 4(k_2-k_1)\left(1-\frac{x_m}{l_f}\right) \Rightarrow$$

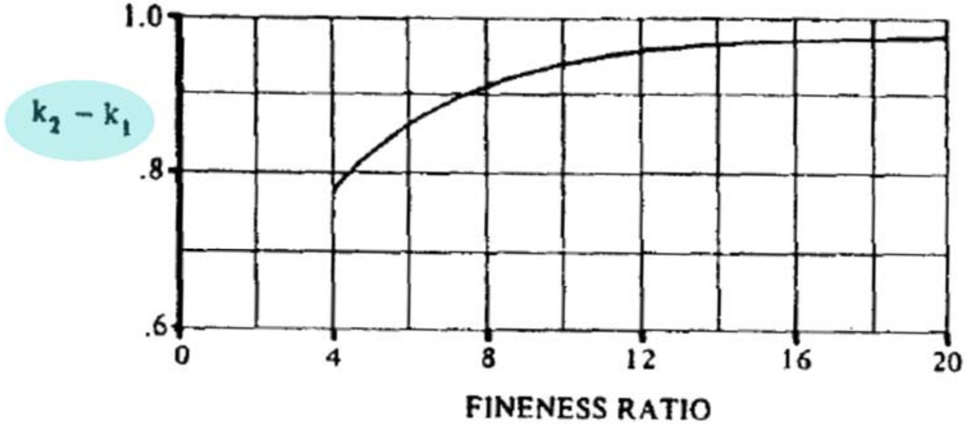


Fig A5 Fig. 3.6 Fuselage apparent mass coefficient.[1]

Contribución Horizontal C_{L_q}

Tail contribution

$$C_{L_{q_h}} = 2C_{L_{h_\alpha}} \eta_h \bar{V}_h$$ ⇨

$C_{L_{h_\alpha}}$ → horizontal tail lift curve slope.
η_h → horizontal tail dynamic pressure ratio.
$\bar{V}_h$ → horizontal tail volume coefficient.

$$\bar{V}_h = \left(\bar{x}_{ac_h} - \bar{x}_{cg}\right)\frac{S_h}{S_w}$$

$\bar{x}_{ac_h}$ → X-location of horizontal tail aerodynamic center in terms of wing mean geometric chord
$\bar{x}_{cg}$ → X-location of the airplane center of gravity in terms of the wing mean geometric chord.
S_h → horizontal tail area.
S_w → wing area.

Contribución V-Tail C_{L_q}

V-Tail contribution

$$C_{L_{q_{vee}}} = 2C_{L_{vee_\alpha}} \eta_{vee} \bar{V}_{vee} \Rightarrow$$

$C_{L_{vee_\alpha}}$ → V-tail lift curve slope.

η_{vee} → V-tail dynamic pressure ratio.

$\bar{V}_{vee}$ → V-tail volume coefficient.

$$\bar{V}_{vee} = \left(\bar{x}_{ac_{vee}} - \bar{x}_{cg}\right)\frac{S_{vee}}{S_w}$$

$\bar{x}_{ac_{vee}}$ → X-location of V-tail aerodynamic center in terms of wing mean geometric chord
$\bar{x}_{cg}$ → X-location of the airplane center of gravity in terms of the wing mean geometric chord.
S_{vee} → V-tail tail area.
S_w → wing area.

Contribución Canard C_{L_q}

Canard contribution

$$C_{L_{q_C}} = -2C_{L_{C_\alpha}}\eta_C\bar{V}_C$$ ⇨

$C_{L_{c\alpha}}$ → canard lift curve slope

η_c → canard dynamic pressure ratio.

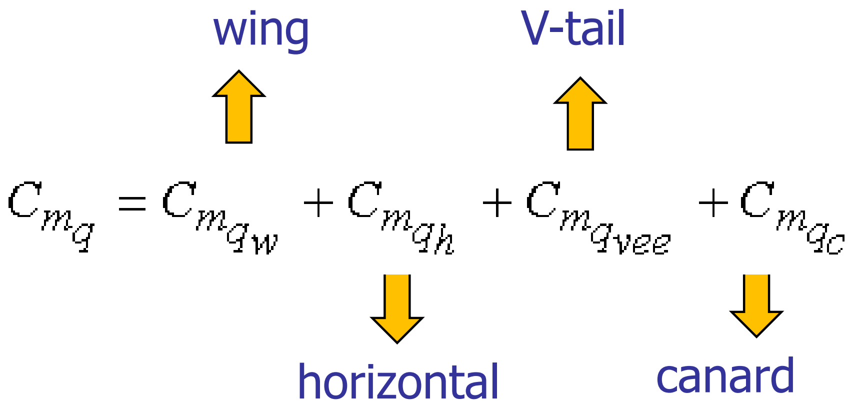
$\bar{V}_c$ → canard volume coefficient.

$$\bar{V}_C = \left(\bar{x}_{ac_C} + \bar{x}_{cg}\right)\frac{S_C}{S_W}$$

$\bar{x}_{ac_c}$ → X-location of canard aerodynamic center in terms of wing mean geometric chord
$\bar{x}_{cg}$ → X-location of the airplane center of gravity in terms of the wing mean geometric chord.
S_h → canard area.
S_w → wing area.

Pitch Moment Derivatives C_{M_q}

The airplane pitching-moment-coefficient-due-to-pitch-rate derivative is



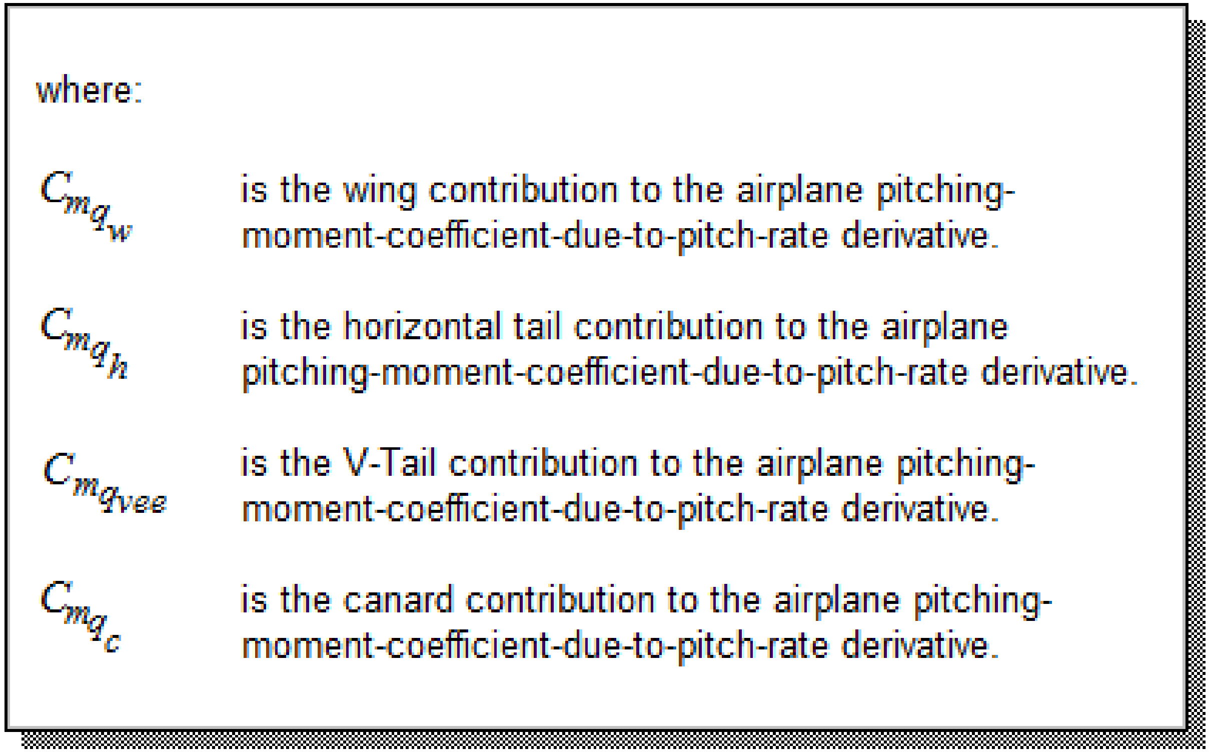
wing

V-tail

$$C_{m_q} = C_{m_{q_w}} + C_{m_{q_h}} + C_{m_{q_{vee}}} + C_{m_{q_c}}$$

horizontal

canard



where:

$C_{m_{q_w}}$	is the wing contribution to the airplane pitching-moment-coefficient-due-to-pitch-rate derivative.
$C_{m_{q_h}}$	is the horizontal tail contribution to the airplane pitching-moment-coefficient-due-to-pitch-rate derivative.
$C_{m_{q_{vee}}}$	is the V-Tail contribution to the airplane pitching-moment-coefficient-due-to-pitch-rate derivative.
$C_{m_{q_c}}$	is the canard contribution to the airplane pitching-moment-coefficient-due-to-pitch-rate derivative.

Contribución Ala C_{M_q}

Wing contribution

Método 1

$$C_{m_{q_w}} = C_{m_{q_w @ M=0}} \left(\frac{\dfrac{AR_w^3 \tan^2 \Lambda_{c/4_w}}{AR_w B + 6\cos\Lambda_{c/4_w}} + \dfrac{3}{B}}{\dfrac{AR_w^3 \tan^2 \Lambda_{c/4_w}}{AR_w + 6\cos\Lambda_{c/4_w}} + 3} \right)$$

AR_w → wing aspect ratio
B → compressibility sweep correction factor
$\Lambda_{c/4_w}$ → is the wing quarter chord angle.

$C_{m_{q_w @ M=0}}$ → wing contribution to airplane pitch-moment- coefficient-due-to-pitch-rate derivative at Mach=0o

$$B = \sqrt{1 - M_1^2 \cos\left(\Lambda_{c/4_w}\right)^2}$$

Contribución Ala C_{M_q}

Wing contribution

Método 1

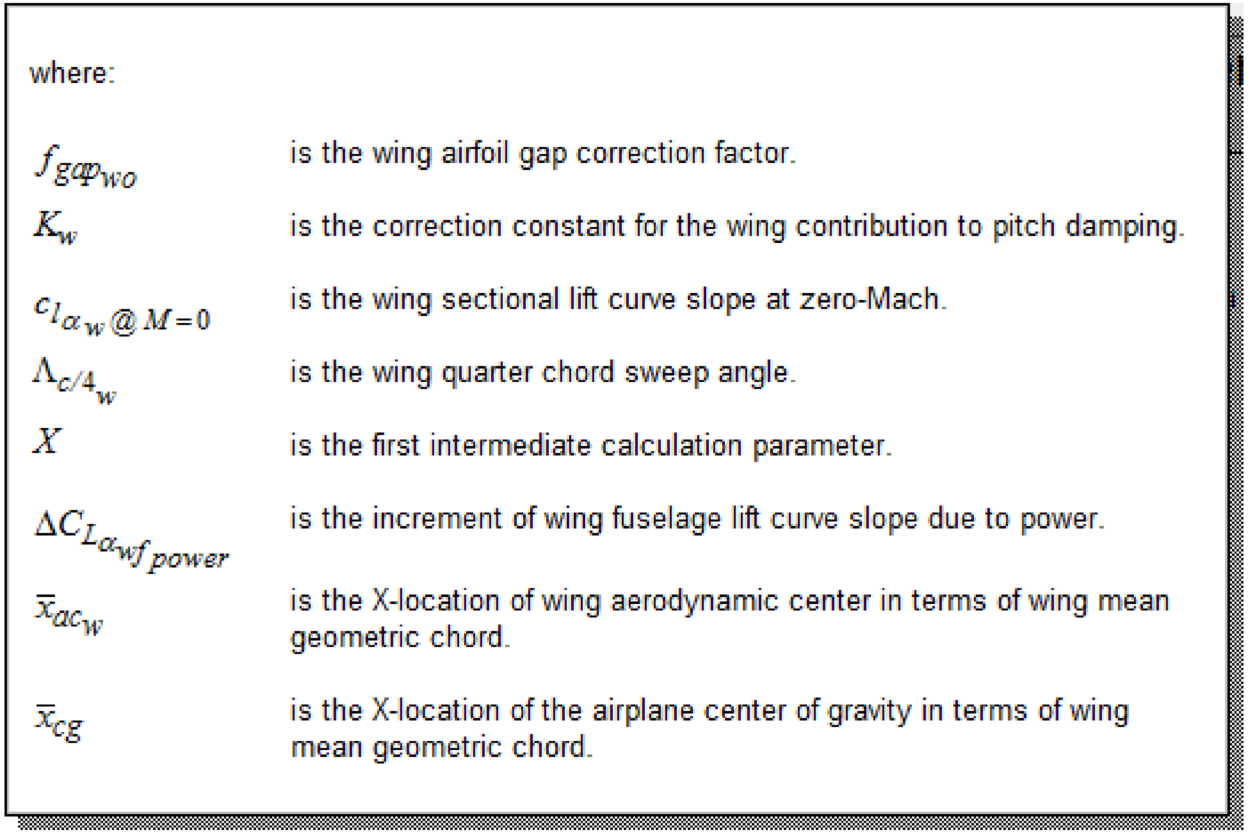
$$C_{m_{q_w @ M=0}} = -f_{gap_{wo}} K_w c_{l_{\alpha_w @ M=0}} \cos \Lambda_{c/4_w} \left[X\right] - 2 \Delta C_{L_{\alpha_{wf_{power}}}} \left(\bar{x}_{ac_w} - \bar{x}_{cg}\right)^2$$

$$C_{l_{aw@M=0}} \rightarrow C_{L_\alpha} \text{ @ } 2D$$

Aproximación $f_{gap_{w0}} \approx 1$

For surfaces with small gap effects	1.00
For surfaces with large gap effects	0.85

1ª Aproximación → $\Delta C_{L_{\alpha_{wf_{power}}}} \approx 0$



where:

$f_{gap_{wo}}$	is the wing airfoil gap correction factor.
K_w	is the correction constant for the wing contribution to pitch damping.
$c_{l_{\alpha_w @ M=0}}$	is the wing sectional lift curve slope at zero-Mach.
$\Lambda_{c/4_w}$	is the wing quarter chord sweep angle.
X	is the first intermediate calculation parameter.
$\Delta C_{L_{\alpha_{wf_{power}}}}$	is the increment of wing fuselage lift curve slope due to power.
$\bar{x}_{ac_w}$	is the X-location of wing aerodynamic center in terms of wing mean geometric chord.
$\bar{x}_{cg}$	is the X-location of the airplane center of gravity in terms of wing mean geometric chord.

Contribución Ala C_{M_q}

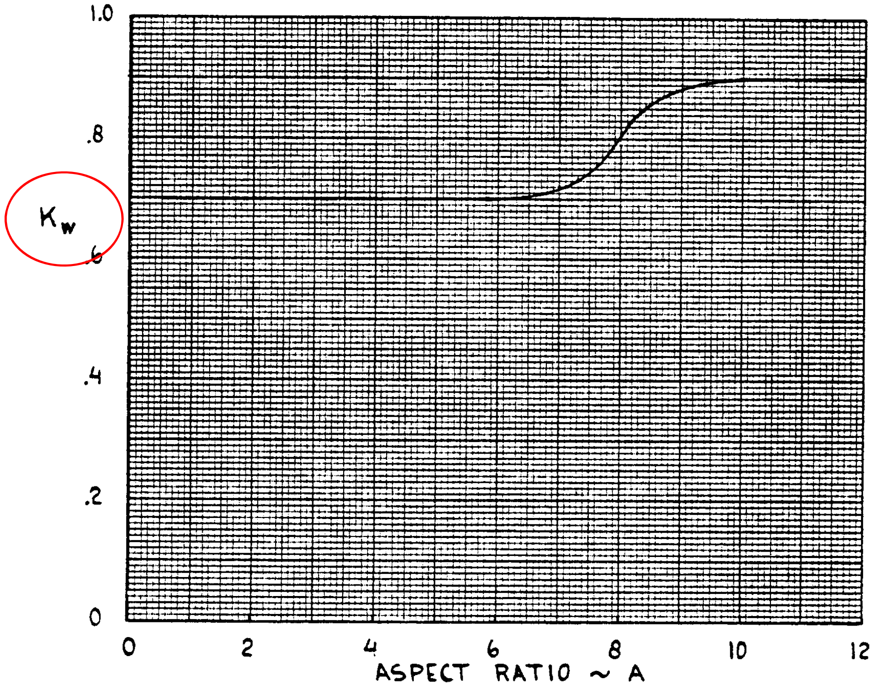
Método 1

The intermediate calculation parameter, X, is given by

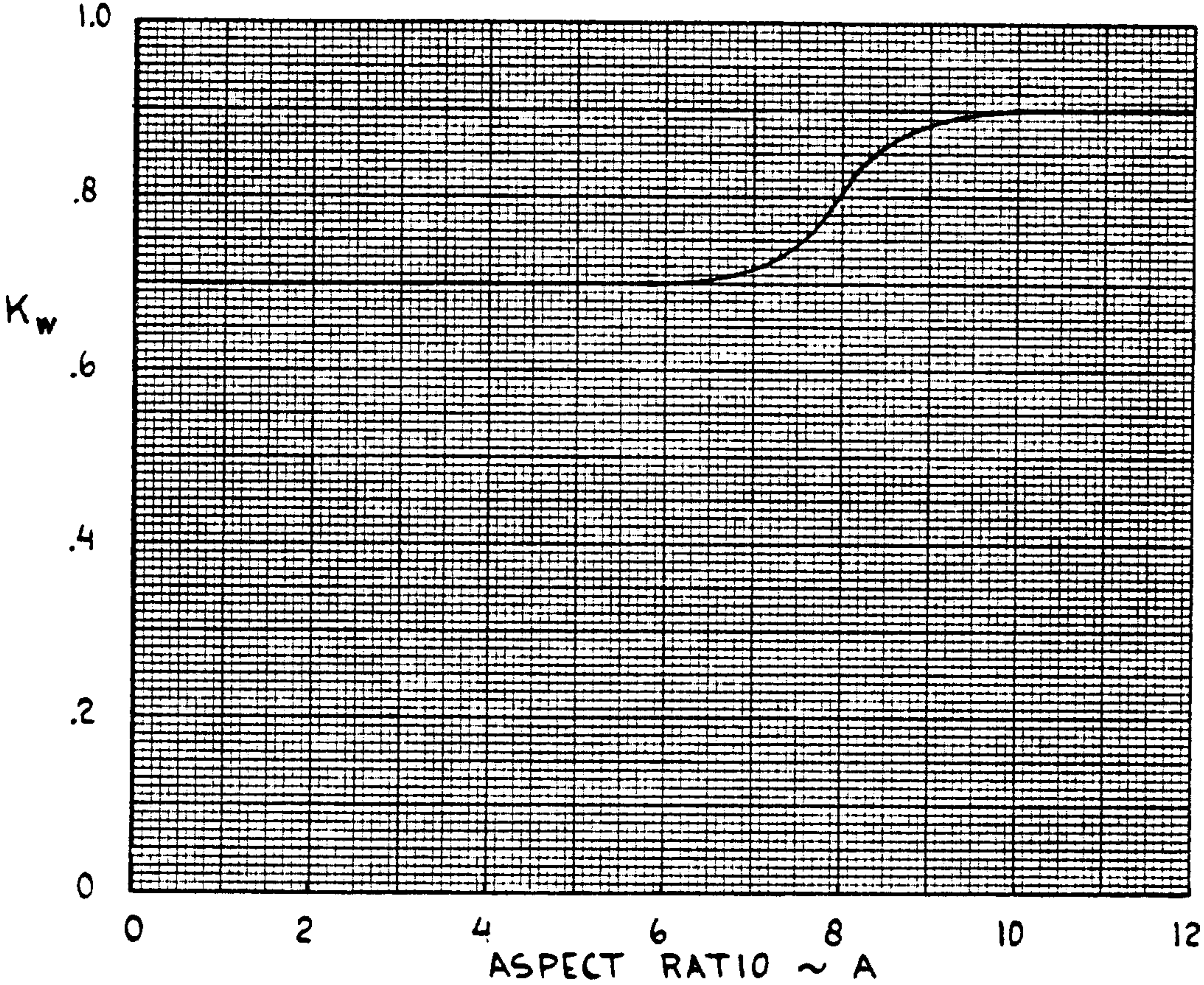
$$X = \frac{1}{8} + \frac{AR_w^3 \tan^2 \Lambda_{c/4_w}}{24\left(AR_w + 6\cos\Lambda_{c/4_w}\right)} + \frac{AR_w\left\{2\left(\bar{x}_{ac_w} - \bar{x}_{cg}\right)^2 + \frac{1}{2}\left(\bar{x}_{ac_w} - \bar{x}_{cg}\right)\right\}}{AR_w + 2\cos\Lambda_{c/4_w}}$$

K_w The correction constant for the wing contribution to pitch damping is obtained from Figure 10.40 in *Airplane Design Part VI* and is a function of the wing aspect ratio:

$$K_w = f\left(AR_w\right)$$



Contribución Ala C_{M_q}



$K_w = f(AR_w)$

Contribución Ala C_{M_q}

The contribution of the wing-body combination

Método 2

$$(C_{mq})_{WB} = [K_{W(B)} + K_{B(W)}]\frac{S_e}{S}\left(\frac{\bar{c}_e}{\bar{c}}\right)^2 (C_{mq})_e + (C_{mq})_B \frac{S_{B,\max}}{S}\left(\frac{l_f}{\bar{c}}\right)^2 \Big/ \text{rad}$$

c_e mean aerodynamic chords of the exposed wing
c mean aerodynamic chords of the total (theoretical) wing
l_f fuselage length
$\left(C_{M_q}\right)_e$ and $(C_M)_B$ → contributions of the exposed wing and isolated body

Velocidades subsónicas ➡

$$(C_{mq})_e = \left[\frac{\frac{c_1}{c_3} + c_2}{\frac{c_1}{c_4} + 3}\right](C_{mq})_{e,M=0.2}$$

$$(C_{mq})_{e,M=0.2} = -0.7 C_{l\alpha} \cos\Lambda_{c/4}\left[\frac{A(0.5\xi + 2\xi^2)}{c_5} + \left(\frac{c_1}{24c_4}\right) + \frac{1}{8}\right]$$

➡

$$\xi = \frac{\bar{x}}{\bar{c}_e}$$

$$\bar{x} = (x_{ac})_e - x_{cg,le}$$

$(x_{ac})_e$ → distance of exposed wing aerodynamic center from the leading edge of the root chord
$x_{cg_{le}}$ → distance of the center of gravity from the leading edge of the exposed wing root chord.
$(x_{ac})_e$ and $x_{cg_{le}}$ are measured parallel to the exposed wing root chord.
The parameter $\bar{x}$ will be positive if the aerodynamic center of the exposed wing $(x_{ac})_e$ is aft of the center of gravity

Contribución Ala C_{M_q}

The contribution of the wing-body combination

Método 2

$$(C_{mq})_e = \left[\frac{\frac{c_1}{c_3} + c_2}{\frac{c_1}{c_4} + 3}\right](C_{mq})_{e,M=0.2}$$

$$c_1 = A^3 \tan^2 \Lambda_{c/4} \qquad c_2 = \frac{3}{B} \qquad c_3 = AB + 6 \cos \Lambda_{c/4}$$

$$c_4 = A + 6 \cos \Lambda_{c/4} \qquad c_5 = A + 2 \cos \Lambda_{c/4}$$

$$B = \sqrt{1 - M^2 \cos^2 \Lambda_{c/4}}$$

$A = A_e$ the aspect ratio of the exposed wing and
C_{l_α} is the sectional or two-dimensional lift-curve slope of the wing

Wing-Fuselage Contribution C_{M_q}

Método 2

$$(C_{mq})_{WB} = [K_{W(B)} + K_{B(W)}]\frac{S_e}{S}\left(\frac{\bar{c}_e}{\bar{c}}\right)^2 (C_{mq})_e + (C_{mq})_B \frac{S_{B,\max}}{S}\left(\frac{l_f}{\bar{c}}\right)^2 \Big/ \text{rad}$$

$$K_{W(B)} = 0.1714\left(\frac{b_{f,\max}}{b}\right)^2 + 0.8326\left(\frac{b_{f,\max}}{b}\right) + 0.9974$$

$$K_{B(W)} = 0.7810\left(\frac{b_{f,\max}}{b}\right)^2 + 1.1976\left(\frac{b_{f,\max}}{b}\right) + 0.0088$$

b_{max} → maximum width of the fuselage

b → wing span.

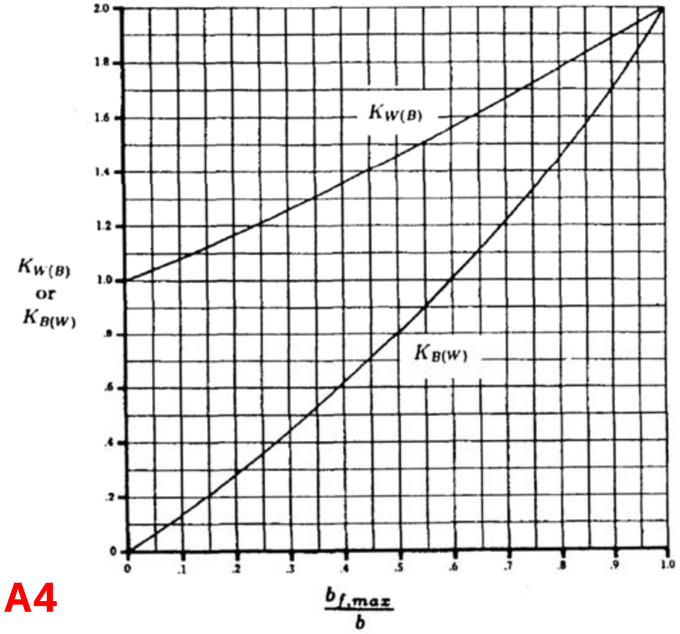
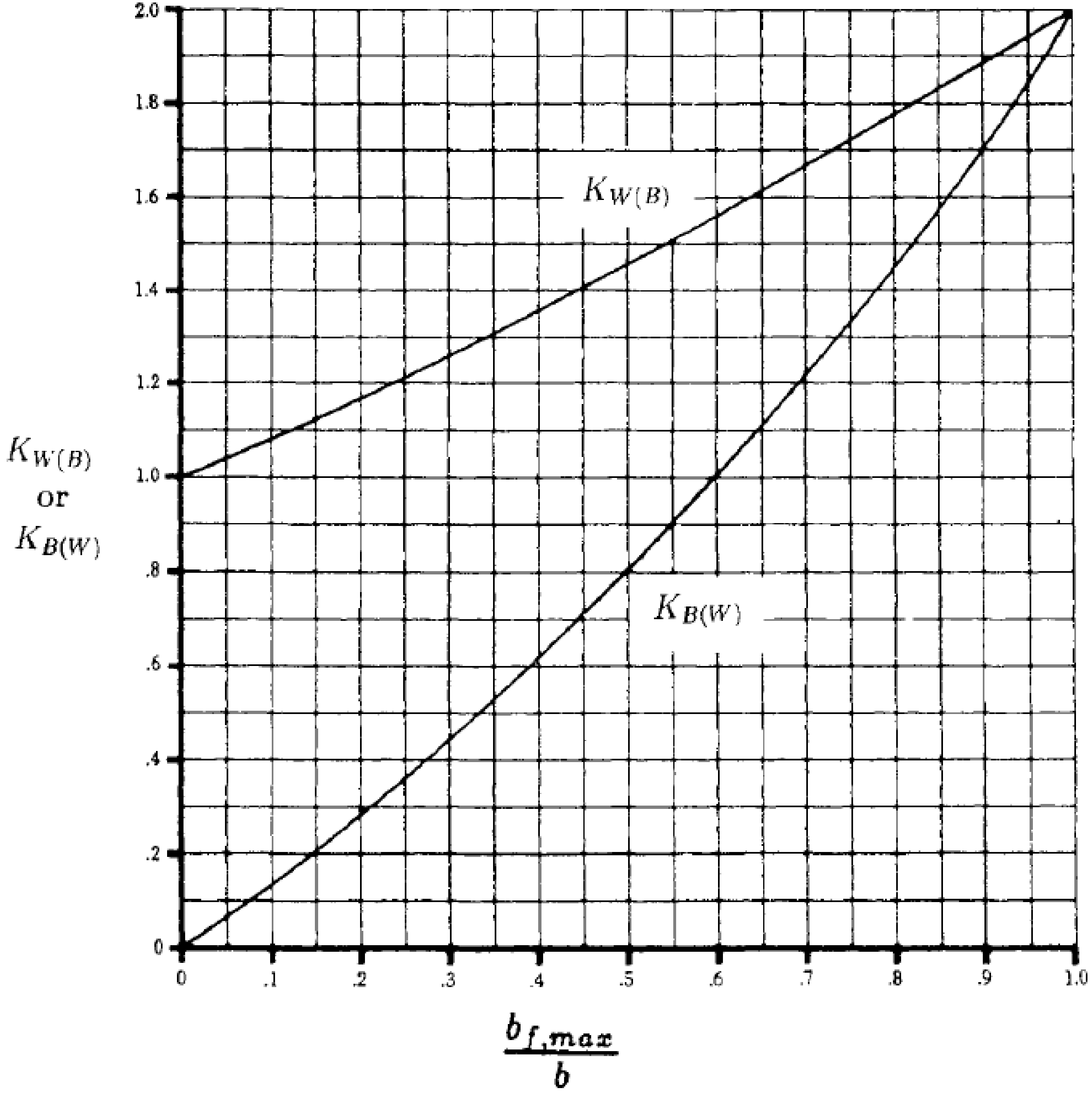


Fig A4

Fig. 3.17 Lift ratios $K_{B(W)}$ and $K_{W(B)}$ (Ref. 1).

Fig A4



Método 2

Contribución Ala C_{M_q}

Método 2

The body contribution

$$(C_{mq})_{WB} = [K_{W(B)} + K_{B(W)}]\frac{S_e}{S}\left(\frac{\bar{c}_e}{\bar{c}}\right)^2 (C_{mq})_e + (C_{mq})_B \frac{S_{B,\max}}{S}\left(\frac{l_f}{\bar{c}}\right)^2 / \text{rad}$$

$$(C_{mq})_B = 2\left(C'_{m\alpha}\right)_B\left[\frac{(1-x_{m1})^2 - V_{B1}(x_{c1}-x_{m1})}{1-x_{m1}-V_{B1}}\right] \Rightarrow \left(C'_{m\alpha}\right)_B = (C_{m\alpha})_B\left(\frac{V_B}{S_{B,\max}l_f}\right)$$

$$x_{m1} = \frac{x_m}{l_f} \qquad x_{c1} = \frac{x_c}{l_f} \qquad V_{B1} = \frac{V_B}{S_{B,\max}l_f} \qquad x_c = \frac{1}{V_B}\int_0^{l_f} S_B(x)x\,\mathrm{d}x$$

$x_m = x_{cg}$ → the distance of the moment reference point from the leading edge of the fuselage,
x_0 is the axial location where the fluid flow over the fuselage ceases to be potential.
$k_2 - k_1$ is the apparent mass constant
$S_{B,max}$ is the maximum cross-sectional area of the fuselage,
l_f total length of the fuselage
V_B volume of the fuselage.

$$(C_{m\alpha})_B = \frac{2(k_2-k_1)}{V_B}\int_0^{x_o}\frac{\mathrm{d}S_B(x)}{\mathrm{d}x}(x_m - x)\,\mathrm{d}x \Rightarrow$$

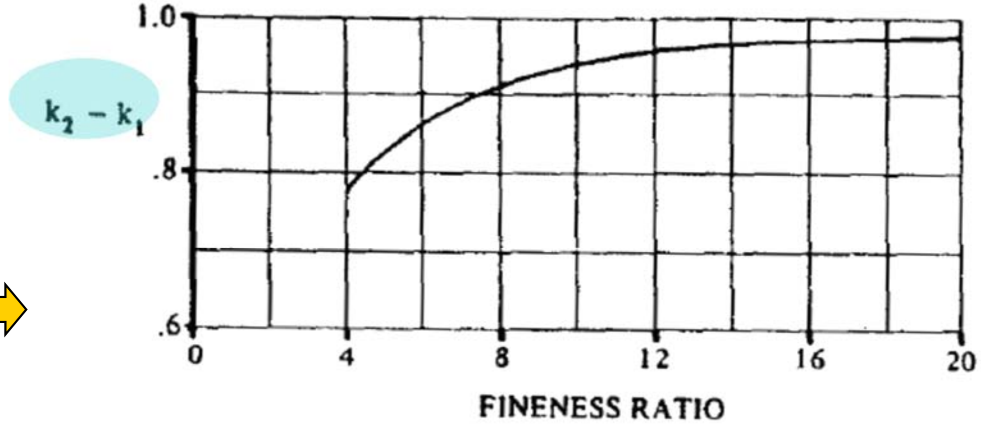


Fig A5

Fig. 3.6 Fuselage apparent mass coefficient.[1]

Contribución Horizontal C_{M_q}

Tail contribution

$$C_{L_{q_h}} = 2C_{L_{h_\alpha}} \eta_h \bar{V}_h$$

$$C_{m_{q_h}} = -2C_{L_{h_\alpha}} \eta_h \bar{V}_h \left(\bar{x}_{ac_h} - \bar{x}_{cg} \right) \Rightarrow$$

$C_{L_{h_\alpha}}$ → horizontal tail lift curve slope.

η_h → horizontal tail dynamic pressure ratio.

$\bar{V}_h$ → horizontal tail volume coefficient

$$\bar{V}_h = \left(\bar{x}_{ac_h} - \bar{x}_{cg} \right) \frac{S_h}{S_w}$$

$\bar{x}_{ac_h}$ → X-location of horizontal tail aerodynamic center in terms of wing mean geometric chord
$\bar{x}_{cg}$ → X-location of the airplane center of gravity in terms of the wing mean geometric chord.
S_h → horizontal tail area.
S_w → wing area.

Contribución V-Tail C_{M_q}

V-Tail contribution

$$C_{m_{q_{vee}}} = -2C_{L_{vee_\alpha}} \eta_{vee} \bar{V}_{vee} \left(\bar{x}_{ac_{vee}} - \bar{x}_{cg} \right) \Rightarrow$$

$C_{L_{vee_\alpha}}$ → V-tail lift curve slope.
η_{vee} → V-tail dynamic pressure ratio.
$\bar{V}_{vee}$ → V-tail volume coefficient.

$$\bar{V}_{vee} = \left(\bar{x}_{ac_{vee}} - \bar{x}_{cg} \right) \frac{S_{vee}}{S_w}$$

$\bar{x}_{ac_{vee}}$ → X-location of V-tail aerodynamic center in terms of wing mean geometric chord
$\bar{x}_{cg}$ → X-location of the airplane center of gravity in terms of the wing mean geometric chord.
S_{vee} → V-tail tail area.
S_w → wing area.

Contribución Canard C_{M_q}

Canard contribution

$$C_{m_{q_c}} = -2C_{L_{c_\alpha}}\eta_c\bar{V}_c\left(\bar{x}_{ac_c}+\bar{x}_{cg}\right)$$

$C_{L_{c\alpha}}$ → canard lift curve slope

η_c → canard dynamic pressure ratio.

$\bar{V}_c$ → canard volume coefficient.

$$\bar{V}_c = \left(\bar{x}_{ac_c}+\bar{x}_{cg}\right)\frac{S_c}{S_w}$$

$\bar{x}_{ac_c}$ → X-location of canard aerodynamic center in terms of wing mean geometric chord
$\bar{x}_{cg}$ → X-location of the airplane center of gravity in terms of the wing mean geometric chord.
S_h → canard area.
S_w → wing area.

Derivadas $C_{D_{\dot{\alpha}}}, C_{L_{\dot{\alpha}}}, C_{M_{\dot{\alpha}}}$

Angle of Attack Rate Derivatives

Estimación Derivadas

- Contribución $C_{D_{\dot{\alpha}}}$

- Contribución $C_{L_{\dot{\alpha}}}$

- Contribución $C_{M_{\dot{\alpha}}}$

Derivadas en 1/rad si no se indica lo contrario
Si las derivadas no están en 1/rad hay que convertirlas

Angle of Attack Rate Derivatives se suelen despreciar en 1ª aproximación

Angle of Attack Rate Derivatives $C_{D_{\dot{\alpha}}}$

The airplane drag-coefficient-due-to-angle-of-attack-rate derivative is normally neglected:

$$C_{D_{\dot{\alpha}}} \approx 0$$

Angle of Attack Rate Derivatives $C_{L_{\dot{\alpha}}}$

Método 1

The airplane lift-coefficient-due-angle-of-attack-rate derivative is determined from

V-tail

$$C_{L_{q_h}} = 2C_{L_{h_\alpha}}\eta_h\bar{V}_h$$

$$C_{L_{\dot{\alpha}}} = C_{L_{\dot{\alpha}_h}} + C_{L_{\dot{\alpha}_{vee}}} + C_{L_{\dot{\alpha}_c}}$$

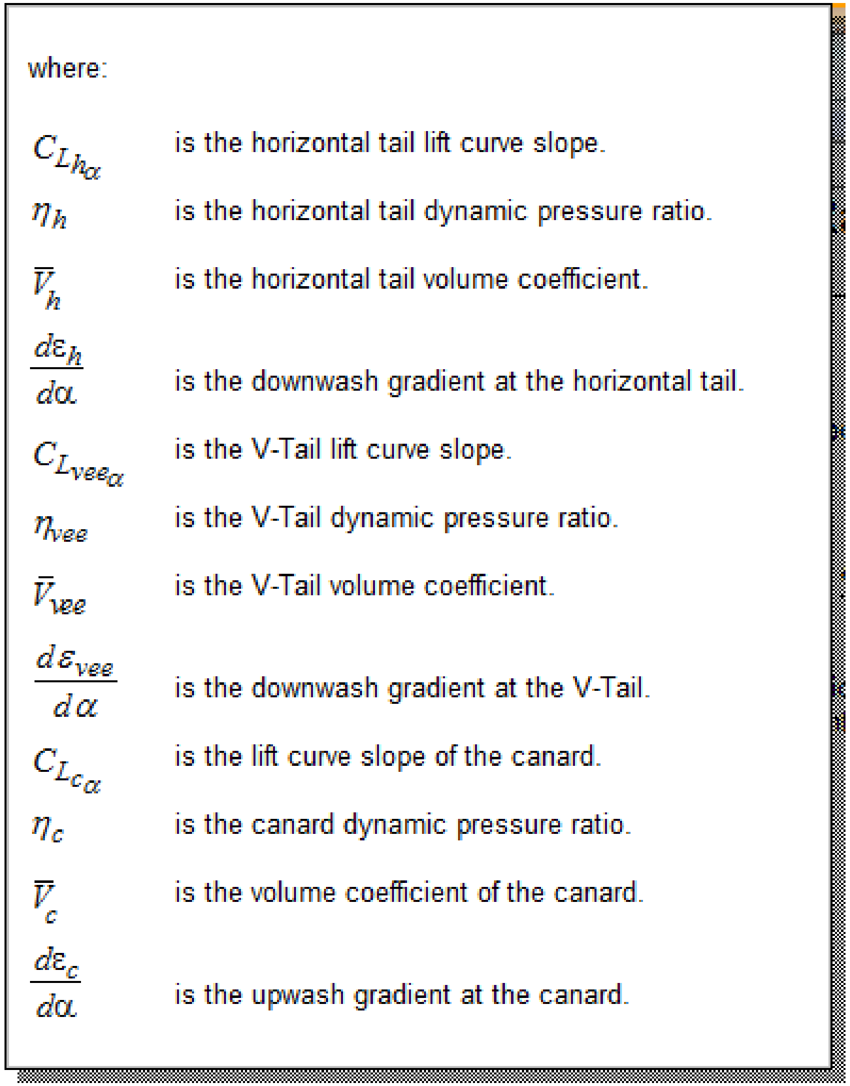
horizontal

canard

horizontal
$$C_{L_{\dot{\alpha}_h}} = 2C_{L_{h_\alpha}}\eta_h\bar{V}_h\frac{d\epsilon_h}{d\alpha}$$

V-tail
$$C_{L_{\dot{\alpha}_{vee}}} = 2C_{L_{vee_\alpha}}\eta_{vee}\bar{V}_{vee}\frac{d\epsilon_{vee}}{d\alpha}$$

canard
$$C_{L_{\dot{\alpha}_c}} = 2C_{L_{c_\alpha}}\eta_c\bar{V}_c\frac{d\epsilon_c}{d\alpha}$$



where:

$C_{L_{h_\alpha}}$	is the horizontal tail lift curve slope.
η_h	is the horizontal tail dynamic pressure ratio.
$\bar{V}_h$	is the horizontal tail volume coefficient.
$\frac{d\varepsilon_h}{d\alpha}$	is the downwash gradient at the horizontal tail.
$C_{L_{vee_\alpha}}$	is the V-Tail lift curve slope.
η_{vee}	is the V-Tail dynamic pressure ratio.
$\bar{V}_{vee}$	is the V-Tail volume coefficient.
$\frac{d\varepsilon_{vee}}{d\alpha}$	is the downwash gradient at the V-Tail.
$C_{L_{c_\alpha}}$	is the lift curve slope of the canard.
η_c	is the canard dynamic pressure ratio.
$\bar{V}_c$	is the volume coefficient of the canard.
$\frac{d\varepsilon_c}{d\alpha}$	is the upwash gradient at the canard.

The equation above is based on the assumption that the contribution of the horizontal tail, V-Tail, and canard are the only important contributions to this derivative

Angle of Attack Rate Derivatives $C_{L_{\dot{\alpha}}}$

Método 1

$$\bar{V}_h = \left(\bar{x}_{ac_h} - \bar{x}_{cg}\right)\frac{S_h}{S_w} \qquad \bar{V}_{vee} = \left(\bar{x}_{ac_{vee}} - \bar{x}_{cg}\right)\frac{S_{vee}}{S_w} \qquad \bar{V}_c = \left(\bar{x}_{ac_c} + \bar{x}_{cg}\right)\frac{S_c}{S_w}$$

$\bar{V}_h$ → canard volume coefficient $\bar{V}_h$ → canard volume coefficient $\bar{V}_{vee}$ → canard volume coefficient

$\bar{x}_{ac_h}$ → X-location of horizontal tail aerodynamic center in terms of wing mean geometric chord
$\bar{x}_{ac_{vee}}$ → X-location of V-tail aerodynamic center in terms of wing mean geometric chord
$\bar{x}_{ac_c}$ → X−location of canard aerodynamic center in terms of wing mean geometric chord
$\bar{x}_{cg}$ → X-location of the airplane center of gravity in terms of the wing mean geometric chord.
S_h → canard area.
S_{vee} → V−tail area.
S_c → canard area.
S_w → wing area.

Angle of Attack Rate Derivatives $C_{L_{\dot{\alpha}}}$

The contribution of the wing-body combination

Método 2

$$(C_{L\dot{\alpha}})_{WB} = [K_{W(B)} + K_{B(W)}]\left(\frac{S_e\bar{c}_e}{S\bar{c}}\right)(C_{L\dot{\alpha}})_e + (C_{L\dot{\alpha}})_B \frac{S_{B,\max} l_f}{S\bar{c}} \Big/ \text{rad}$$

c_e mean aerodynamic chords of the exposed wing
c mean aerodynamic chords of the total (theoretical) wing
l_f fuselage length
$\left(C_{L_{\dot{\alpha}}}\right)_e$ and $\left(C_{L_{\dot{\alpha}}}\right)_B$ → contributions of the exposed wing and isolated body

Velocidades subsónicas ⇒ $(C_{L\dot{\alpha}})_e = 1.5\left(\frac{x_{ac}}{c_r}\right)_e (C_{L\alpha})_e + 3C_L(g)/\text{rad}$

$$C_L(g) = \left(\frac{-\pi A_e}{2\beta^2}\right)(0.0013\,\tau^4 - 0.0122\,\tau^3 + 0.0317\,\tau^2 + 0.0186\,\tau - 0.0004)$$

$$\tau = \beta A_e, \qquad \beta = \sqrt{1 - M^2}.$$

Angle of Attack Rate Derivatives $C_{L_{\dot{\alpha}}}$

The body contribution

Método 2

$$(C_{L\dot{\alpha}})_{WB} = [K_{W(B)} + K_{B(W)}]\left(\frac{S_e\bar{c}_e}{S\bar{c}}\right)(C_{L\dot{\alpha}})_e + (C_{L\dot{\alpha}})_B\,\frac{S_{B,\max} l_f}{S\bar{c}} \Big/ \text{rad}$$

$$(C_{L\dot{\alpha}})_B = 2(C'_{L\alpha})_B\left(\frac{V_B}{S_{B,\max} l_f}\right)$$

→

$$(C'_{L\alpha})_B = (C_{L\alpha})_B\left(\frac{V_B^{2/3}}{S_{B,\max}}\right)$$

$$(C_{L\alpha})_B = 2(k_2 - k_1)\left(\frac{S_{B,\max}}{V_B^{2/3}}\right)$$

$k_2 - k_1$ is the apparent mass constant

$S_{B,max}$ is the maximum cross-sectional area of the fuselage,

l_f total length of the fuselage

V_B volume of the fuselage.

$$(C_{L_{\dot{\alpha}}})_B = 4(k_2 - k_1)\left(\frac{V_B}{S_{B,max} l_f}\right)$$

→

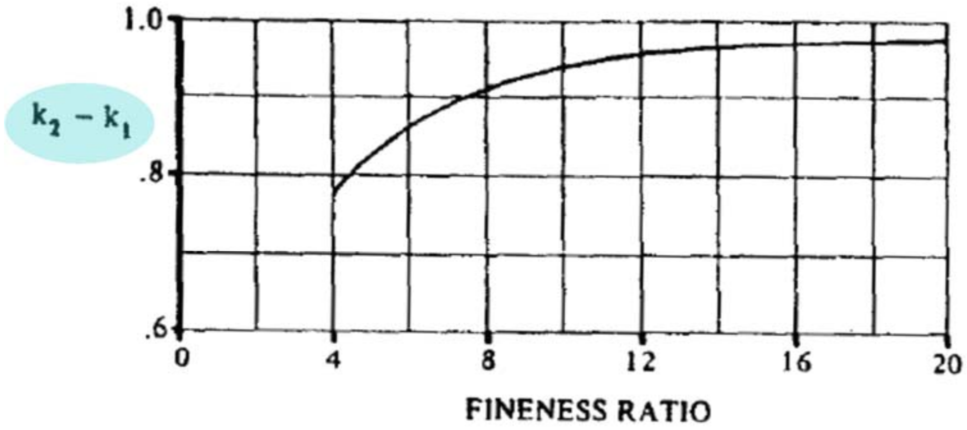


Fig A5 Fig. 3.6 Fuselage apparent mass coefficient.[1]

Angle of Attack Rate Derivatives $C_{M_{\dot{\alpha}}}$

Método 1

The airplane pitching-moment-coefficient-due-angle-of-attack-rate derivative is determined from

V-tail

$$C_{m_{\dot{\alpha}}} = C_{m_{\dot{\alpha}_h}} + C_{m_{\dot{\alpha}_{vee}}} + C_{m_{\dot{\alpha}_c}}$$

horizontal

canard

The equation above is based on the assumption that the contribution of the horizontal tail, V-Tail, and canard are the only important contributions to this derivative

horizontal ⇨ $C_{m_{\dot{\alpha}_h}} = -C_{L_{\dot{\alpha}_h}}\left(\bar{x}_{ac_h} - \bar{x}_{cg}\right)$ ⇨ $C_{L_{\dot{\alpha}_h}} = 2C_{L_{h_\alpha}}\eta_h\bar{V}_h\dfrac{d\epsilon_h}{d\alpha}$

V-tail ⇨ $C_{m_{\dot{\alpha}_{vee}}} = -C_{L_{\dot{\alpha}_{vee}}}\left(\bar{x}_{ac_{vee}} - \bar{x}_{cg}\right)$ ⇨ $C_{L_{\dot{\alpha}_{vee}}} = 2C_{L_{vee_\alpha}}\eta_{vee}\bar{V}_{vee}\dfrac{d\epsilon_{vee}}{d\alpha}$

canard ⇨ $C_{m_{\dot{\alpha}_c}} = -C_{L_{\dot{\alpha}_c}}\left(\bar{x}_{ac_c} + \bar{x}_{cg}\right)$ ⇨ $C_{L_{\dot{\alpha}_c}} = 2C_{L_{c_\alpha}}\eta_c\bar{V}_c\dfrac{d\epsilon_c}{d\alpha}$

Angle of Attack Rate Derivatives $C_{M_{\dot{\alpha}}}$

Método 1

The equation above is based on the assumption that the contribution of the horizontal tail, V-Tail, and canard are the only important contributions to this derivative

$$\bar{V}_h = \left(\bar{x}_{ac_h} - \bar{x}_{cg}\right)\frac{S_h}{S_w} \qquad \bar{V}_{vee} = \left(\bar{x}_{ac_{vee}} - \bar{x}_{cg}\right)\frac{S_{vee}}{S_w} \qquad \bar{V}_c = \left(\bar{x}_{ac_c} + \bar{x}_{cg}\right)\frac{S_c}{S_w}$$

$\bar{V}_h$ → canard volume coefficient $\bar{V}_h$ → canard volume coefficient $\bar{V}_{vee}$ → canard volume coefficient

$\bar{x}_{ac_h}$ → X-location of horizontal tail aerodynamic center in terms of wing mean geometric chord
$\bar{x}_{ac_{vee}}$ → X-location of V-tail aerodynamic center in terms of wing mean geometric chord
$\bar{x}_{ac_c}$ → X−location of canard aerodynamic center in terms of wing mean geometric chord
$\bar{x}_{cg}$ → X-location of the airplane center of gravity in terms of the wing mean geometric chord.
S_h → canard area.
S_{vee} → V−tail area.
S_c → canard area.
S_w → wing area.

Angle of Attack Rate Derivatives $C_{M_{\dot{\alpha}}}$

The contribution of the wing-body combination

Método 2

$$(C_{m\dot{\alpha}})_{WB} = [K_{W(B)} + K_{B(W)}]\left(\frac{S_e\bar{c}_e^2}{S\bar{c}^2}\right)(C_{m\dot{\alpha}})_e + (C_{m\dot{\alpha}})_B\frac{S_{B,\max}l_f^2}{S\bar{c}^2} \Big/ \text{rad}$$

c_e mean aerodynamic chords of the exposed wing
c mean aerodynamic chords of the total (theoretical) wing
l_f fuselage length
$\left(C_{M_{\dot{\alpha}}}\right)_e$ and $\left(C_{M_{\dot{\alpha}}}\right)_B$ → contributions of the exposed wing and isolated body

Velocidades subsónicas ⇨ $(C_{m\dot{\alpha}})_e = (C''_{m\dot{\alpha}})_e + \left(\frac{x_{cg,le}}{\bar{c}_e}\right)(C_{L\dot{\alpha}})_e \Big/ \text{rad}$

$$(C''_{m\dot{\alpha}})_e = -\left(\frac{81}{32}\right)\left(\frac{x_{ac}}{c_r}\right)_e^2(C_{L\alpha})_e + \frac{9}{2}C_{mo}(g) \Big/ \text{rad}$$

$$(C_{L\dot{\alpha}})_e = 1.5\left(\frac{x_{ac}}{c_r}\right)_e(C_{L\alpha})_e + 3C_L(g)/\text{rad}$$

Angle of Attack Rate Derivatives $C_{M_{\dot\alpha}}$

The contribution of the wing-body combination

Método 2

Velocidades subsónicas ⇒ $(C_{m\dot\alpha})_e = (C''_{m\dot\alpha})_e + \left(\frac{x_{cg,le}}{\bar{c}_e}\right)(C_{L\dot\alpha})_e \Big/ \text{rad}$

$$(C''_{m\dot\alpha})_e = -\left(\frac{81}{32}\right)\left(\frac{x_{ac}}{c_r}\right)_e^2 (C_{L\alpha})_e + \frac{9}{2}C_{m_o}(g) \Big/ \text{rad}$$

$$(C_{L\dot\alpha})_e = 1.5\left(\frac{x_{ac}}{c_r}\right)_e (C_{L\alpha})_e + 3C_L(g)/\text{rad}$$

⇒ $(C_{m\dot\alpha})_B = 2(C'_{m\alpha})_B\left[\frac{x_{c1} - x_{m1}}{1 - x_{m1} - V_{B1}}\right]\left(\frac{V_B}{S_{B,\max}l_f}\right)$

$$(C'_{m\alpha})_B = (C_{m\alpha})_B\left(\frac{V_B}{S_{B,\max}l_f}\right)$$

$$x_{m1} = \frac{x_m}{l_f} \qquad x_{c1} = \frac{x_c}{l_f} \qquad V_{B1} = \frac{V_B}{S_{B,\max}l_f} \qquad x_c = \frac{1}{V_B}\int_0^{l_f} S_B(x)x\,dx$$

$x_m = x_{cg}$ → the distance of the moment reference point from the leading edge of the fuselage,
x_0 is the axial location where the fluid flow over the fuselage ceases to be potential.
$k_2 - k_1$ is the apparent mass constant
$S_{B,max}$ is the maximum cross-sectional area of the fuselage,
l_f total length of the fuselage
V_B volume of the fuselage.

Derivadas $C_{T_{x_1}}, C_{T_{x_u}}$ $C_{M_{T_1}}$ $C_{M_{T_u}}$ $C_{T_{x\alpha}}$ $C_{M_{T\alpha}}$

Propulsive Derivatives

Propulsive Derivatives $C_{T_{x_1}}$

The airplane steady state thrust coefficient is defined as:

$$C_{T_{x_1}} = \frac{T_{set}\cos(\phi_T + \alpha)}{\overline{q}_1 S_w}$$

Steady State Flight

$$C_{T_{x_1}} = C_{D_1}$$

Propulsive Derivatives $C_{T_{x_u}}$

Airplanes with pure jets

$$C_{T_{x_u}} = \left(\frac{U_1}{\bar{q}_1 S_w}\right)\left(2A_{thrust}U_1 + B_{thrust}\right) - 2C_{T_{x_1}}$$

Modelo propulsivo - RFP

$$T = A_{thrust}U_1^2 + B_{thrust}U_1 + C_{thrust}$$

⇒

U_1 → airplane steady state flight speed.
A_{thurst} → A coefficient in thrust vs. speed quadratic equation.
B_{thrust} → B coefficient in thrust vs. speed quadratic equation.
C_{thrust} → C coefficient in thrust vs. speed quadratic equation.

Airplanes with variable pitch propeller driven engines

$$C_{T_{x_u}} = -3C_{T_{x_1}}$$

Airplanes with fixed pitch propeller driven engines

$$C_{T_{x_u}} = \left(\frac{1}{\bar{q}_1 S_w}\right)\left(2A_{power}U_1 + B_{power}\right) - 2C_{T_{x_1}}$$

Modelo propulsivo - RFP

$$P = A_{power}U_1^2 + B_{power}U_1 + C_{power}$$

U_1 → airplane steady state flight speed.
A_{power} → A coefficient in power versus speed quadratic equation.
B_{power} → B coefficient in power versus speed quadratic equation.
C_{power} → C coefficient in power versus speed quadratic equation.

Propulsive Derivatives $C_{M_{T_1}}$

The airplane steady state thrust pitching moment coefficient for a jet airplane is given by:

$$C_{m_{T_1}} = \frac{-T_{avail}\, d_T}{\bar{q}_1 S_w \bar{c}_w}$$

Trim condiitions

$$\Sigma C_m = C_{m_{T_1}} + C_{m_1} = 0$$

$$C_{m_{T_1}} = -C_{m_1}$$

Fig A16

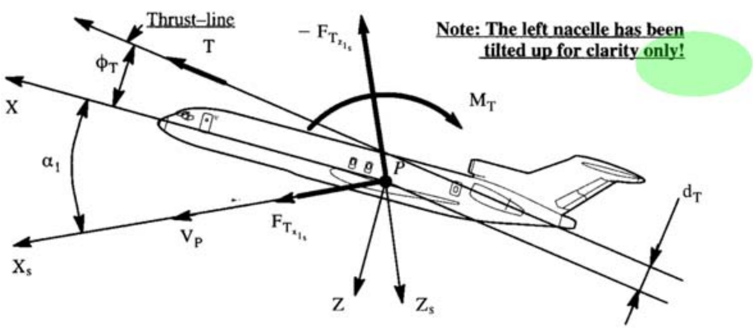
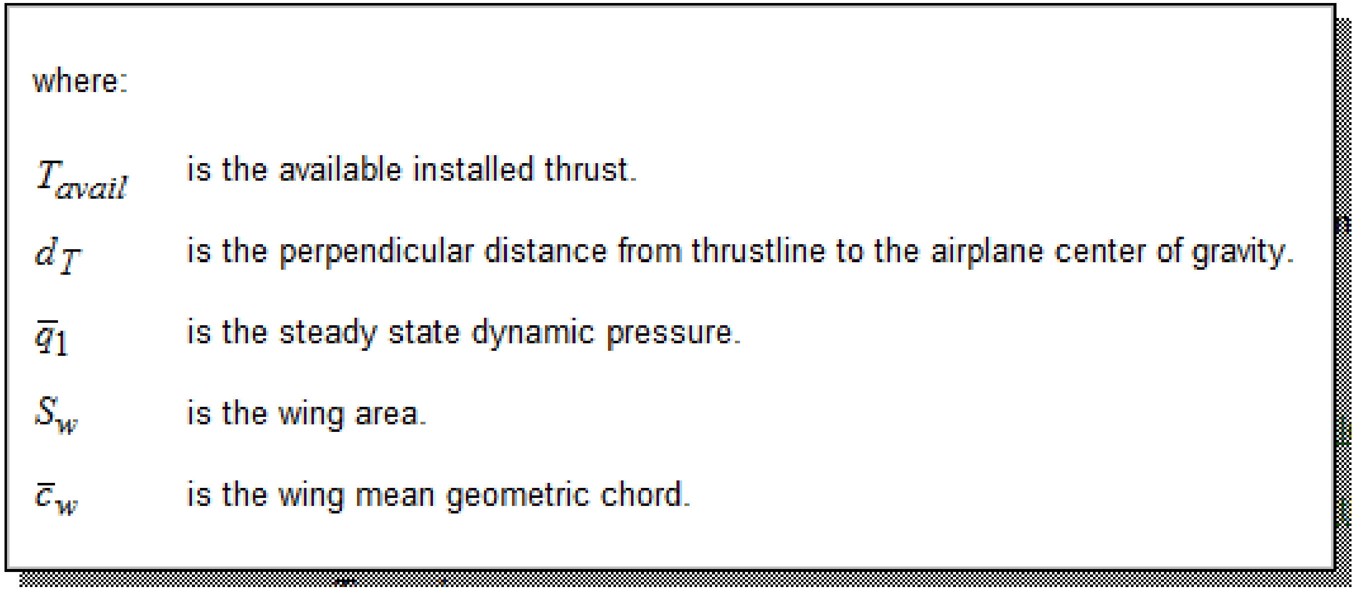


Figure 3.26 Steady State Thrust Forces and Pitching Moment in Stability Axes



where:

T_{avail}	is the available installed thrust.
d_T	is the perpendicular distance from thrustline to the airplane center of gravity.
$\bar{q}_1$	is the steady state dynamic pressure.
S_w	is the wing area.
$\bar{c}_w$	is the wing mean geometric chord.

Fig A16

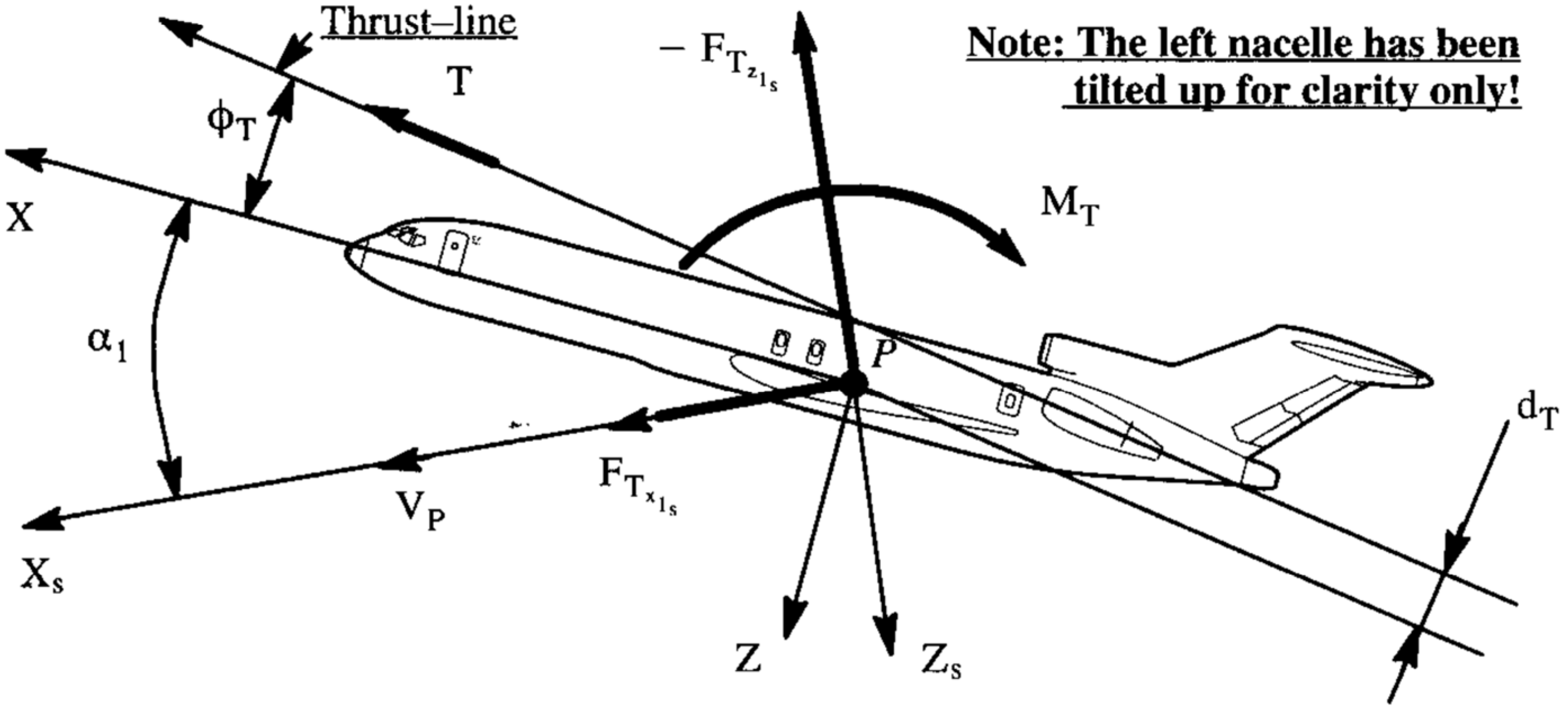


Figure 3.26 Steady State Thrust Forces and Pitching Moment in Stability Axes

Propulsive Derivatives $C_{M_{T_1}}$

The airplane steady state thrust pitching moment coefficient for a propeller airplane is given by:

$$C_{m_{T_1}} = \Delta C_{m_{N_{prop}}} + \Delta C_{m_{T_{prop}}} \Rightarrow \Delta C_{m_{T_{prop}}} = \frac{-T_{avail}\, d_T}{\bar{q}_1 S_w \bar{c}_w}$$

Aproximación → $\Delta C_{m_{T_{prop}}} \approx 0$

For propeller ⇒ $T_{avail} = SHP_{set}\left(1 - K_{loss}\right)\eta_p \dfrac{550}{U_1}$

wing mean geometric chord ⇒ $\bar{c}_w = \dfrac{4}{3}\dfrac{1+\lambda_w+\lambda_w^2}{\left(1+\lambda_w\right)^2}\sqrt{\dfrac{S_w}{AR_w}}$

The perpendicular distance from the thrust line to the airplane center of gravity is found from

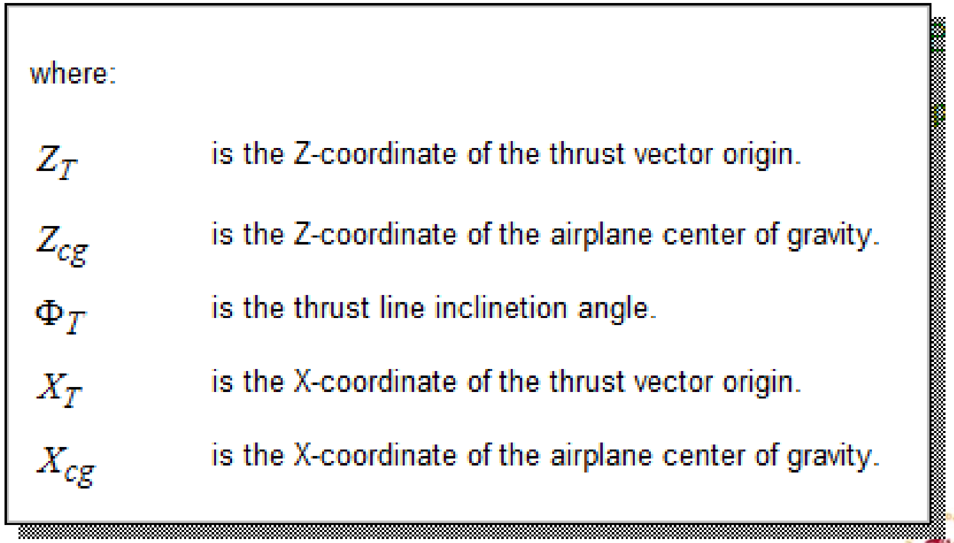
$$d_T = \left(Z_T - Z_{cg}\right)\cos\Phi_T + \left(X_T - X_{cg}\right)\sin\Phi_T$$

Aproximación → $\phi_T \approx 0$

Trim condiitions

$$\Sigma C_m = C_{m_{T_1}} + C_{m_1} = 0$$

$$C_{m_{T_1}} = -C_{m_1}$$



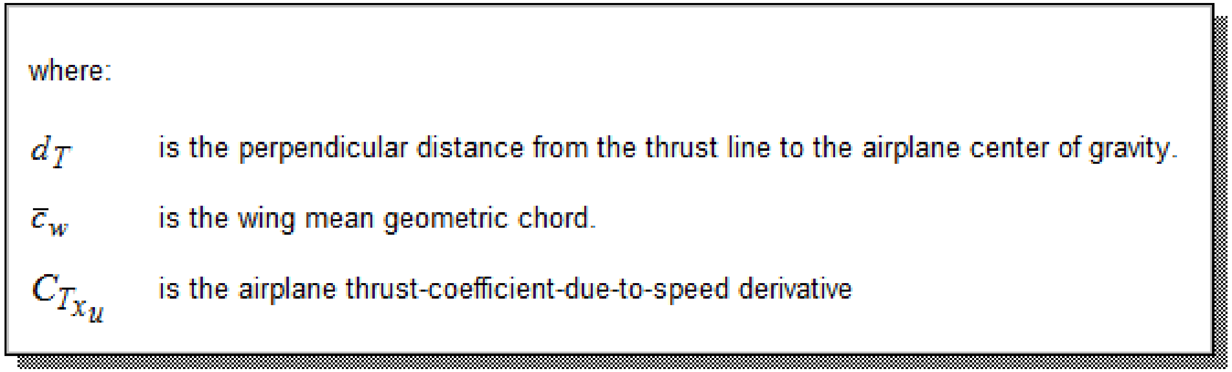
where:

Z_T	is the Z-coordinate of the thrust vector origin.
Z_{cg}	is the Z-coordinate of the airplane center of gravity.
Φ_T	is the thrust line inclinetion angle.
X_T	is the X-coordinate of the thrust vector origin.
X_{cg}	is the X-coordinate of the airplane center of gravity.

Propulsive Derivatives $C_{M_{T_u}}$

The airplane thrust-pitching-moment-coefficient-due-to-speed derivative is defined as the variation of airplane pitching moment coefficient due to thrust with dimensionless speed:

$$C_{m_{T_u}} = -\left(\frac{d_T}{\bar{c}_w}\right) C_{T_{x_u}}$$



where:

d_T	is the perpendicular distance from the thrust line to the airplane center of gravity.
$\bar{c}_w$	is the wing mean geometric chord.
$C_{T_{x_u}}$	is the airplane thrust-coefficient-due-to-speed derivative

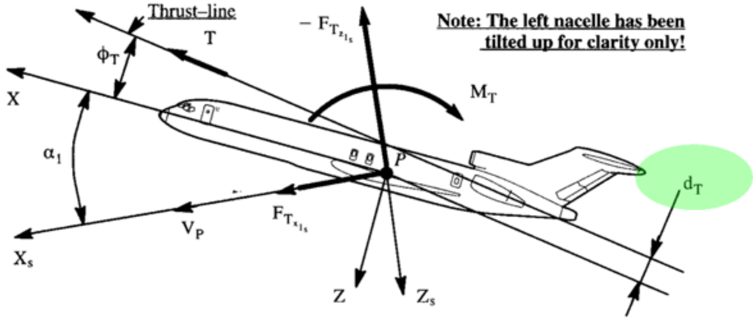


Fig A16

Figure 3.26 Steady State Thrust Forces and Pitching Moment in Stability Axes

Propulsive Derivatives $C_{T_{x_\alpha}}$

The airplane steady state thrust coefficient is defined as:

$$C_{T_{x_\alpha}} \approx 0$$

Propulsive Derivatives $C_{M_{T_\alpha}}$

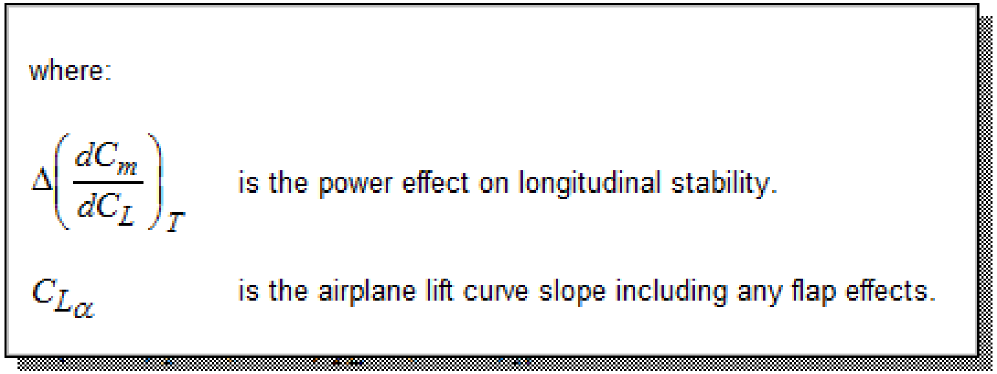
The airplane thrust-pitching-moment-coefficient-due-to-angle-of-attack derivative is defined as

The airplane thust-pitching-moment-coefficient-due-to-angle-of-attack derivative is defined as

$$C_{m_{T_\alpha}} = \frac{\partial C_{m_T}}{\partial \alpha}$$

The airplane thust-pitching-moment-coefficient-due-to-angle-of-attack derivative is defined as

$$C_{m_{T_\alpha}} = \left\{ \Delta \left(\frac{dC_m}{dC_L} \right)_T \right\} C_{L_\alpha}$$



where:

$\Delta \left(\frac{dC_m}{dC_L} \right)_T$	is the power effect on longitudinal stability.
C_{L_α}	is the airplane lift curve slope including any flap effects.

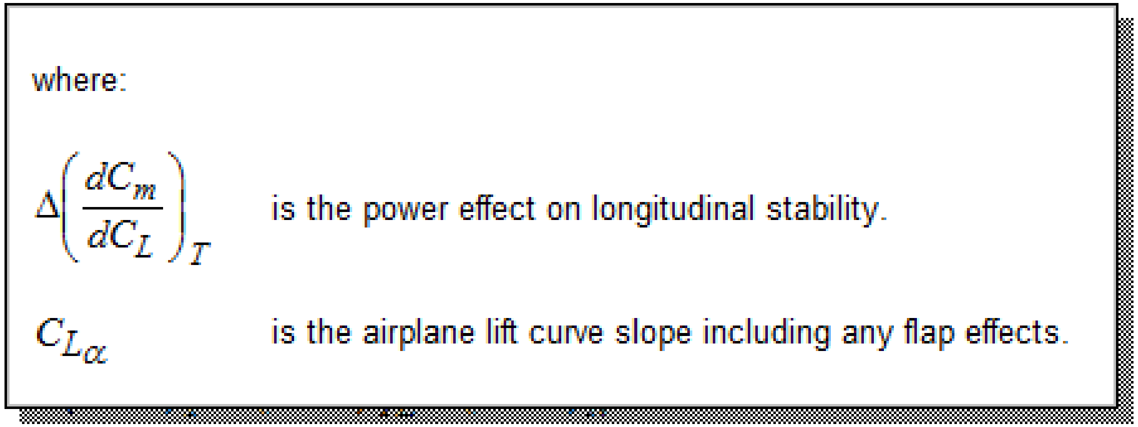
Para aviones jet ⇨ Aproximación $C_{M_{T_\alpha}} \approx 0$

Para aviones con hélice ⇨ Muy compleja estimación ⇨ Aproximación $C_{M_{T_\alpha}} \approx 0$

Propulsive Derivatives $C_{M_{T_\alpha}}$

For propeller driven airplanes:

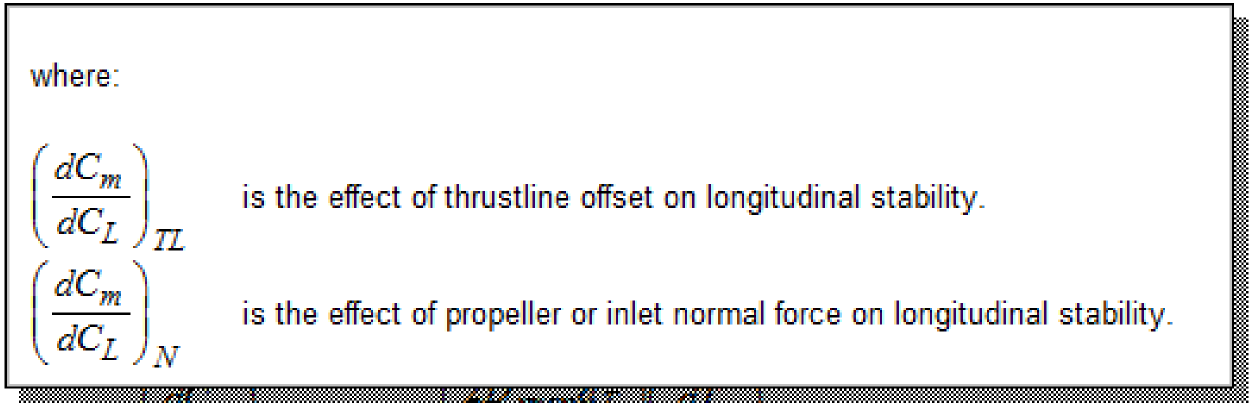
$$C_{m_{T_\alpha}} = \left\{\Delta\left(\frac{dC_m}{dC_L}\right)_T\right\} C_{L_\alpha}$$



where:

$\Delta\left(\frac{dC_m}{dC_L}\right)_T$	is the power effect on longitudinal stability.
C_{L_α}	is the airplane lift curve slope including any flap effects.

$$\Delta\left(\frac{dC_m}{dC_L}\right)_T = \left(\frac{dC_m}{dC_L}\right)_{TL} + \left(\frac{dC_m}{dC_L}\right)_N$$



where:

$\left(\frac{dC_m}{dC_L}\right)_{TL}$	is the effect of thrustline offset on longitudinal stability.
$\left(\frac{dC_m}{dC_L}\right)_N$	is the effect of propeller or inlet normal force on longitudinal stability.

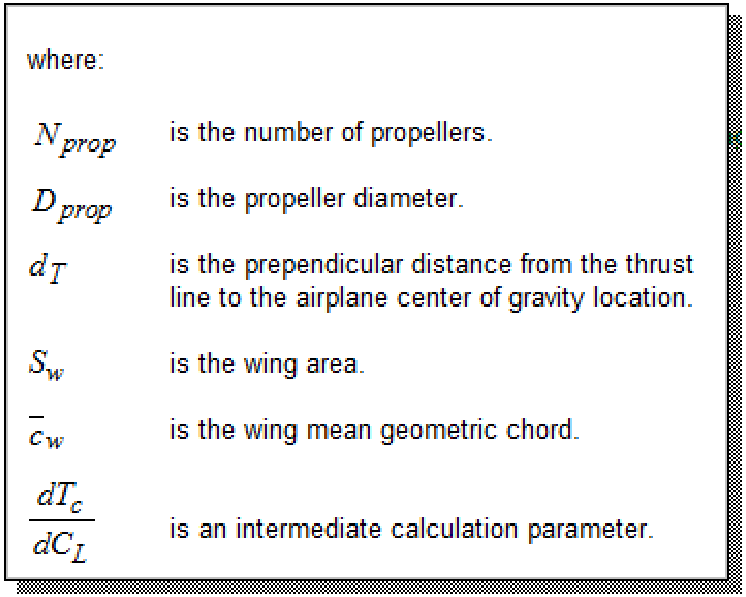
$$\left(\frac{dC_m}{dC_L}\right)_{TL} = N_{prop}\left(\frac{2D_{prop}^2 d_T}{S_w \bar{c}_w}\right)\left(\frac{dT_c}{dC_L}\right)$$

$$\left(\frac{dC_m}{dC_L}\right)_N = \frac{\frac{\pi}{4} f_{inflow} N_{prop} l_{prop} D_{prop}^2 \left(\frac{dC_N}{d\alpha}\right)_{prop}\left(1-\frac{d\varepsilon_u}{d\alpha}\right)}{S_w \bar{c}_w C_{L_\alpha}}$$

Propulsive Derivatives $C_{M_{T_\alpha}}$

For propeller driven airplanes:

$$\left(\frac{dC_m}{dC_L}\right)_{TL} = N_{prop}\left(\frac{2D_{prop}^2 d_T}{S_w \bar{c}_w}\right)\left(\frac{dT_c}{dC_L}\right)$$

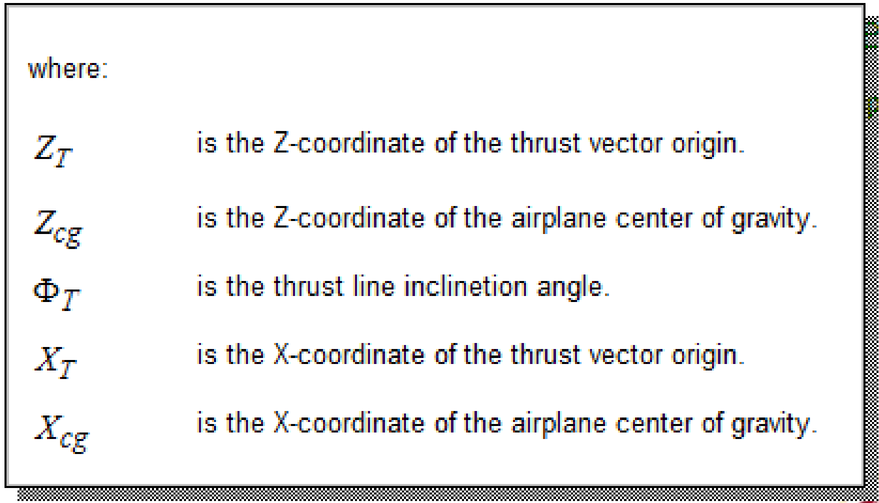


where:

N_{prop}	is the number of propellers.
D_{prop}	is the propeller diameter.
d_T	is the prepnedicular distance from the thrust line to the airplane center of gravity location.
S_w	is the wing area.
$\bar{c}_w$	is the wing mean geometric chord.
$\frac{dT_c}{dC_L}$	is an intermediate calculation parameter.

The perpendicular distance from the thrust line to the airplane center of gravity is found from

$$d_T = \left(Z_T - Z_{cg}\right)\cos\Phi_T + \left(X_T - X_{cg}\right)\sin\Phi_T$$



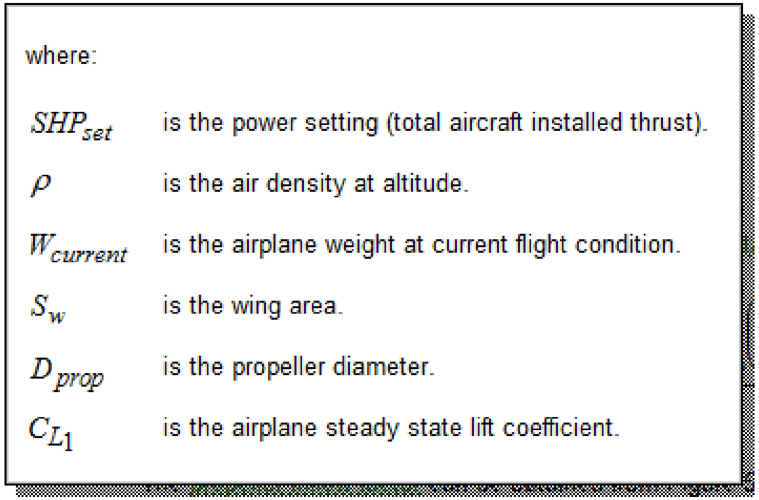
where:

Z_T	is the Z-coordinate of the thrust vector origin.
Z_{cg}	is the Z-coordinate of the airplane center of gravity.
Φ_T	is the thrust line inclinetion angle.
X_T	is the X-coordinate of the thrust vector origin.
X_{cg}	is the X-coordinate of the airplane center of gravity.

Propulsive Derivatives $C_{M_{T\alpha}}$

For propeller driven airplanes:

$$\boxed{\frac{dT_c}{dC_L}} = \frac{3}{2}\frac{550SHP_{set}\sqrt{\rho}}{\sqrt{\left(\frac{2W_{current}}{S_w}\right)^3 D_{prop}^2}}\sqrt{C_{L_1}}$$



where:

SHP_{set}	is the power setting (total aircraft installed thrust).
ρ	is the air density at altitude.
$W_{current}$	is the airplane weight at current flight condition.
S_w	is the wing area.
D_{prop}	is the propeller diameter.
C_{L_1}	is the airplane steady state lift coefficient.

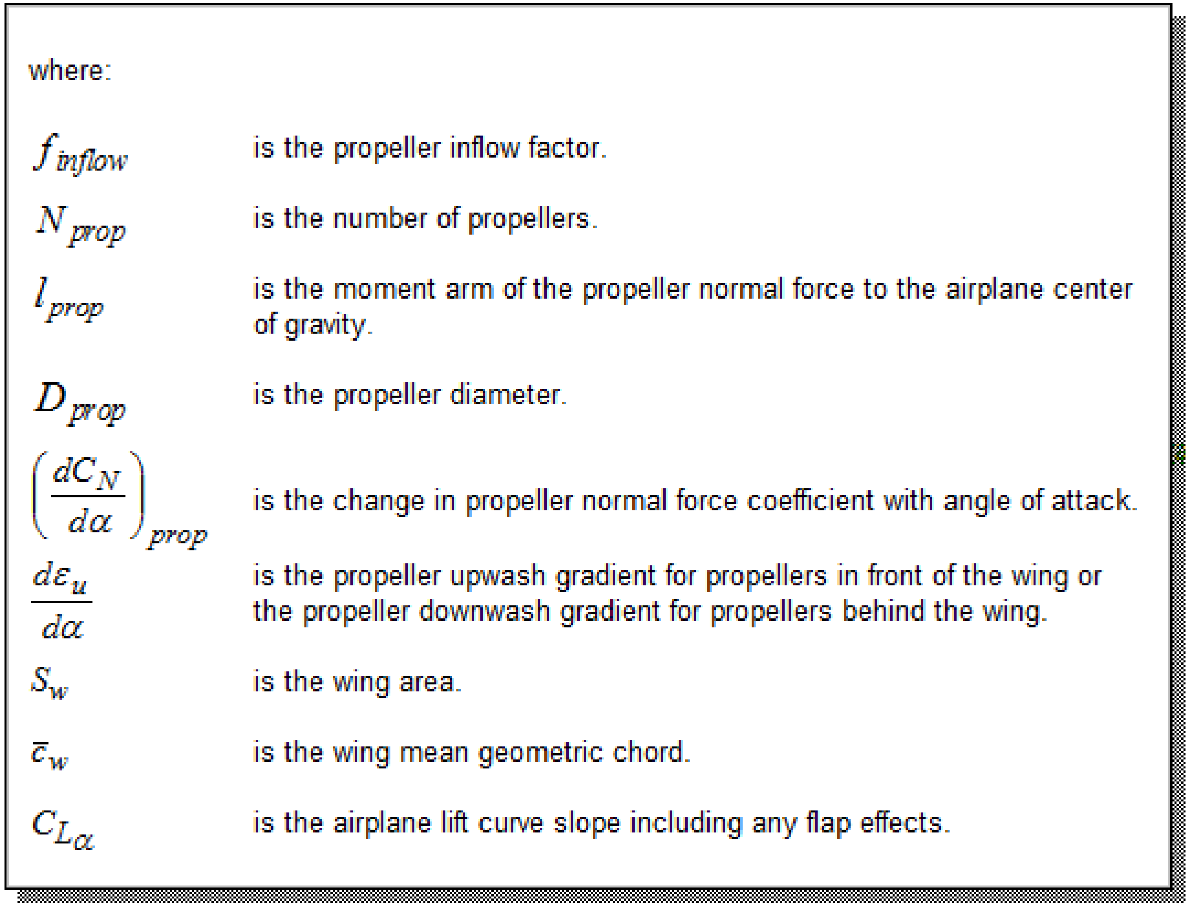
The effect of propeller or inlet normal force on longitudinal stability is given by:

$$\boxed{\left(\frac{dC_m}{dC_L}\right)}_N = \frac{\frac{\pi}{4} f_{inflow} N_{prop} l_{prop} D_{prop}^2 \left(\frac{dC_N}{d\alpha}\right)_{prop}\left(1-\frac{d\varepsilon_u}{d\alpha}\right)}{S_w \bar{c}_w C_{L_\alpha}}$$

Propulsive Derivatives $C_{M_{T\alpha}}$

For propeller driven airplanes:

$$\left(\frac{dC_m}{dC_L}\right)_N = \frac{\frac{\pi}{4} f_{inflow} N_{prop} l_{prop} D_{prop}^2 \left(\frac{dC_N}{d\alpha}\right)_{prop} \left(1 - \frac{d\varepsilon_u}{d\alpha}\right)}{S_w \bar{c}_w C_{L_\alpha}}$$



where:

f_{inflow}	is the propeller inflow factor.
N_{prop}	is the number of propellers.
l_{prop}	is the moment arm of the propeller normal force to the airplane center of gravity.
D_{prop}	is the propeller diameter.
$\left(\frac{dC_N}{d\alpha}\right)_{prop}$	is the change in propeller normal force coefficient with angle of attack.
$\frac{d\varepsilon_u}{d\alpha}$	is the propeller upwash gradient for propellers in front of the wing or the propeller downwash gradient for propellers behind the wing.
S_w	is the wing area.
$\bar{c}_w$	is the wing mean geometric chord.
C_{L_α}	is the airplane lift curve slope including any flap effects.

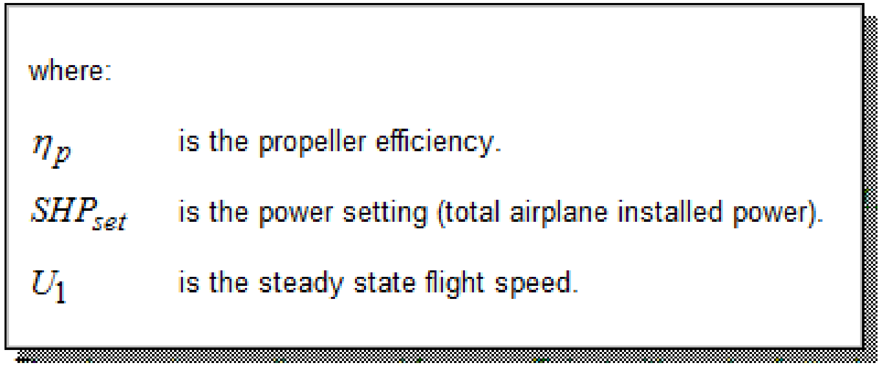
Propulsive Derivatives $C_{M_{T_\alpha}}$

For propeller driven airplanes:

$$f_{inflow} = f\left(T_{set}, \bar{q}_1, D_{prop}\right)$$

Aproximación → $f_{inflow} \approx 1$

$$T_{set} = \frac{550\,\eta_p SHP_{set}}{1.689 U_1}$$

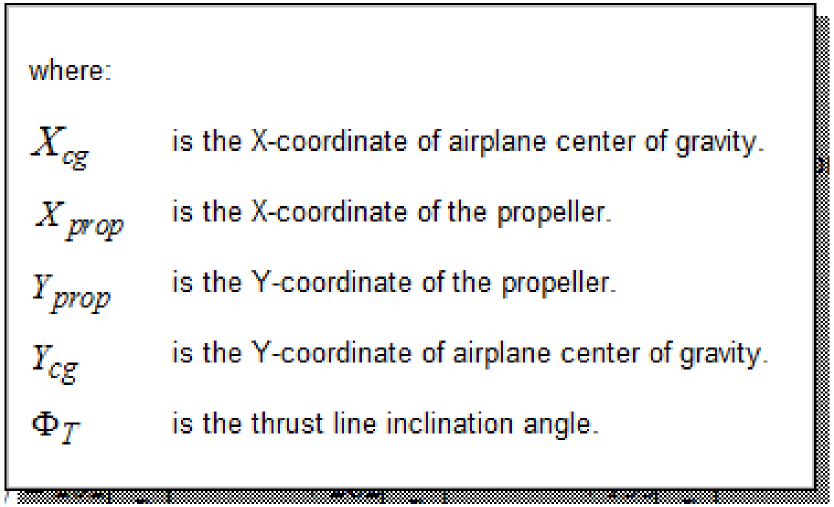


where:

η_p	is the propeller efficiency.
SHP_{set}	is the power setting (total airplane installed power).
U_1	is the steady state flight speed.

The moment arm of the propeller normal force to the airplane center of gravity is given by:

$$l_{prop} = \left(X_{cg} - X_{prop}\right)\cos\Phi_T - \left(Y_{cg} - Y_{prop}\right)\sin\Phi_T$$



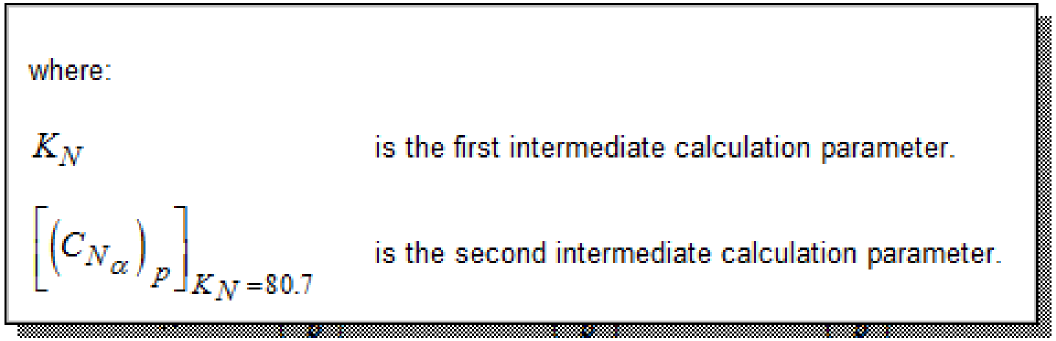
where:

X_{cg}	is the X-coordinate of airplane center of gravity.
X_{prop}	is the X-coordinate of the propeller.
Y_{prop}	is the Y-coordinate of the propeller.
Y_{cg}	is the Y-coordinate of airplane center of gravity.
Φ_T	is the thrust line inclination angle.

Propulsive Derivatives $C_{M_{T_\alpha}}$

For propeller driven airplanes:

$$\left(\frac{dC_N}{d\alpha}\right)_{prop} = \left[\left(C_{N_\alpha}\right)_p\right]_{K_N=80.7}\left\{1+0.8\left[\left(\frac{K_N}{80.7}\right)-1\right]\right\}$$



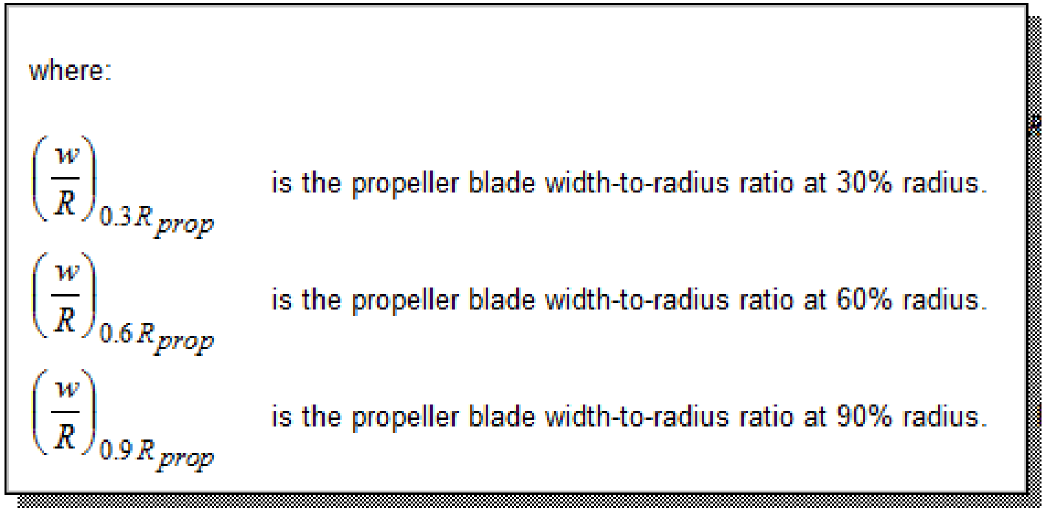
where:

K_N	is the first intermediate calculation parameter.
$\left[\left(C_{N_\alpha}\right)_p\right]_{K_N=80.7}$	is the second intermediate calculation parameter.

The first intermediate calculation parameter is given by

$$K_N = 262\left(\frac{w}{R}\right)_{0.3R_{prop}} + 262\left(\frac{w}{R}\right)_{0.6R_{prop}} + 135\left(\frac{w}{R}\right)_{0.9\,R_{prop}}$$

Geometría de la hélice



where:

$\left(\frac{w}{R}\right)_{0.3R_{prop}}$	is the propeller blade width-to-radius ratio at 30% radius.
$\left(\frac{w}{R}\right)_{0.6R_{prop}}$	is the propeller blade width-to-radius ratio at 60% radius.
$\left(\frac{w}{R}\right)_{0.9R_{prop}}$	is the propeller blade width-to-radius ratio at 90% radius.

The propeller blade radius

$$R_{prop} = \frac{D_{prop}}{2}$$

D_{prop} diameter of prop

Propulsive Derivatives $C_{M_{T_\alpha}}$

For propeller driven airplanes:

$$\left(\frac{dC_N}{d\alpha}\right)_{prop} = \left[\left(C_{N_\alpha}\right)_p\right]_{K_N=80.7}\left\{1+0.8\left[\left(\frac{K_N}{80.7}\right)-1\right]\right\}$$

The second intermediate calculation parameter is obtained from Figure 8.130 in Airplane Design Part VI and is a function of the number of propeller blades and the nominal propeller blade angle at 75% radius.

$$\left[\left(C_{N_\alpha}\right)_p\right]_{K_N=80.7}$$

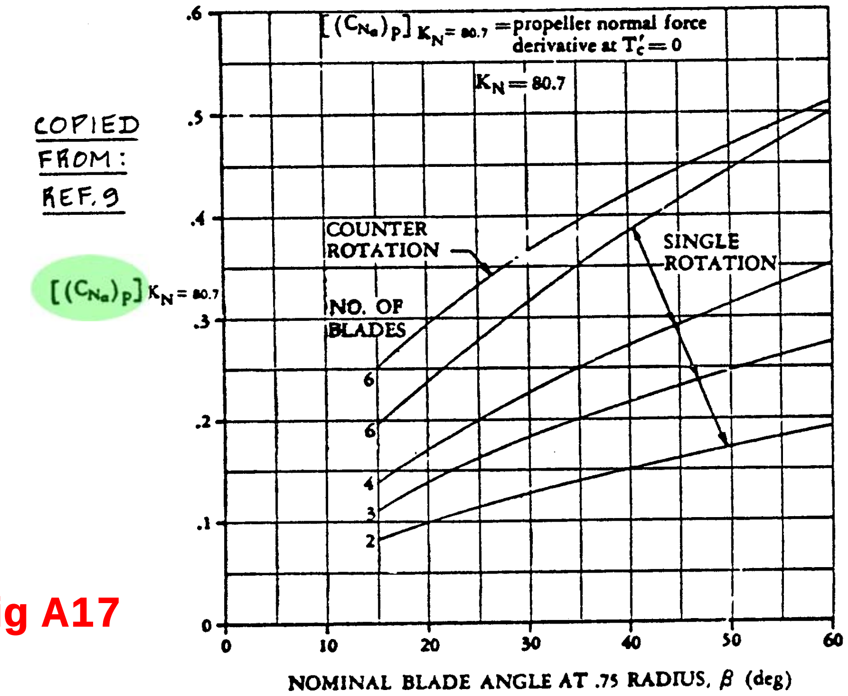


Fig A17

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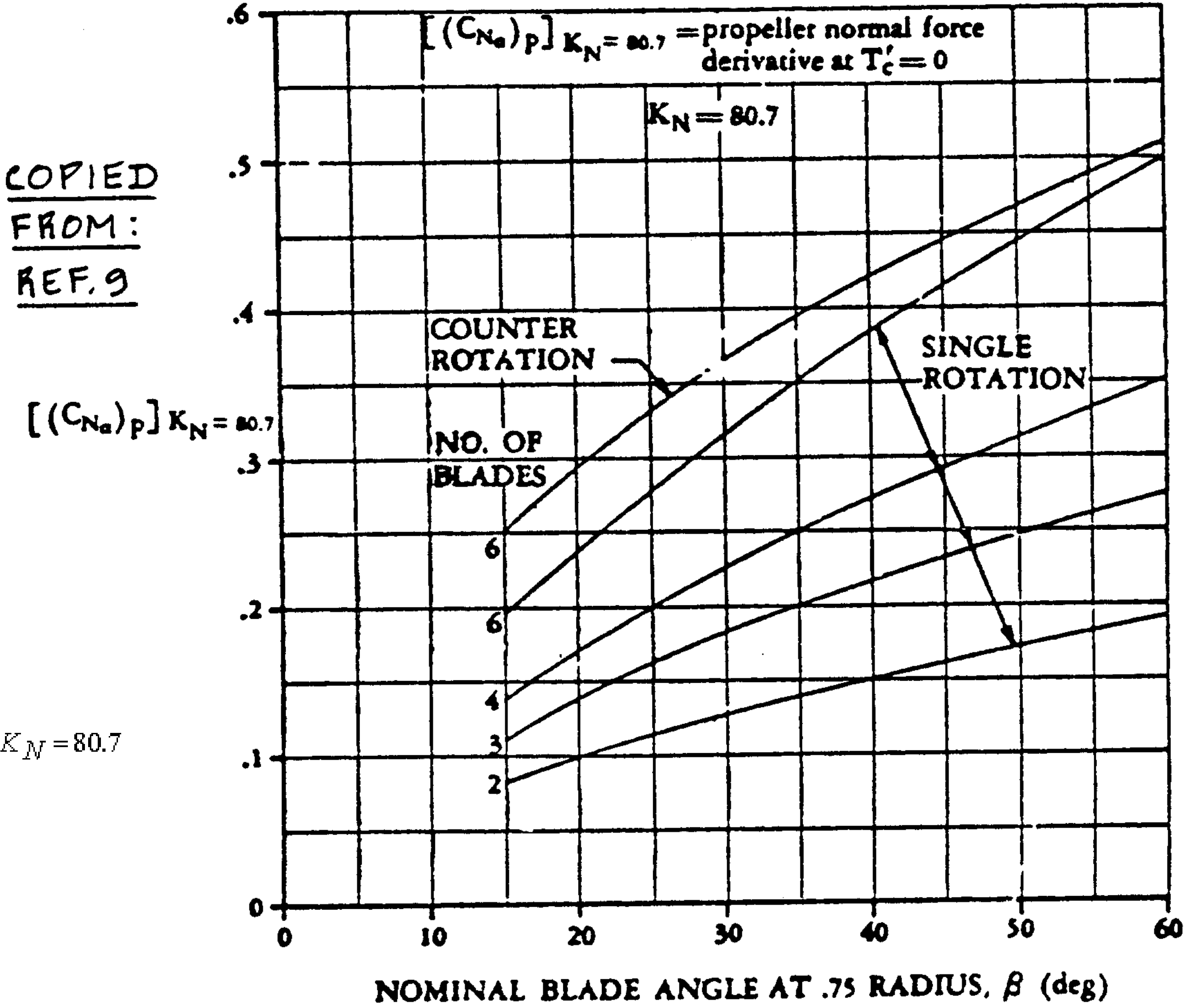


$$\left[\left(C_{N_\alpha}\right)_p\right]_{K_N=80.7}$$

Propulsive Derivatives $C_{M_{T_\alpha}}$

For propeller in front of the wing, the propeller upwash gradient is obtained from Figure 8.67 in Airplane Design Part VI. It is a function of the X-location of the propeller relative to the wing root quarter chord point and the wing aspect ratio

$$\left(\frac{dC_m}{dC_L}\right)_N = \frac{\frac{\pi}{4} f_{inflow} N_{prop} l_{prop} D^2_{prop} \left(\frac{dC_N}{d\alpha}\right)_{prop} \left(1 - \frac{d\varepsilon_u}{d\alpha}\right)}{S_w \bar{c}_w C_{L_\alpha}}$$

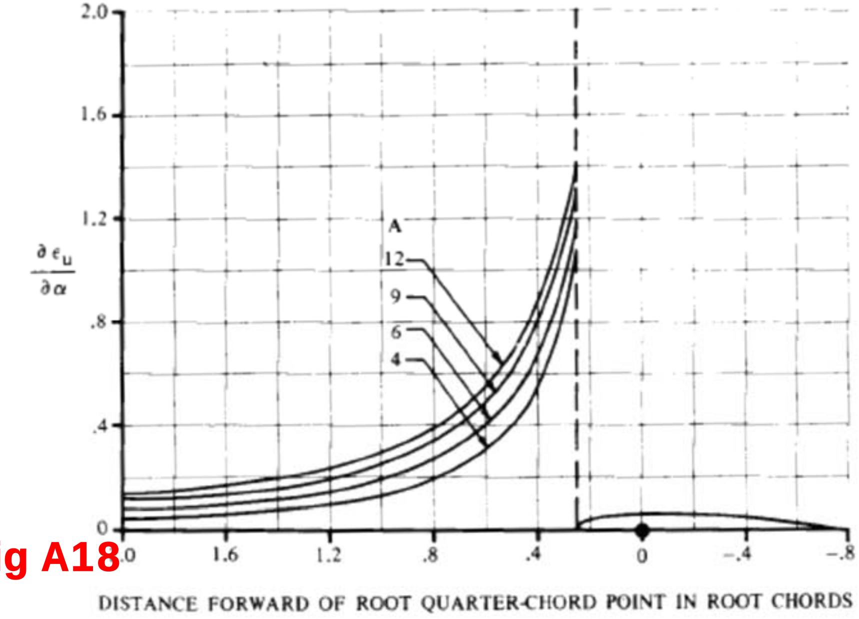


Fig A18

Fig. 16.11 Upwash estimation (subsonic only). (Ref. 37)

Fig A18

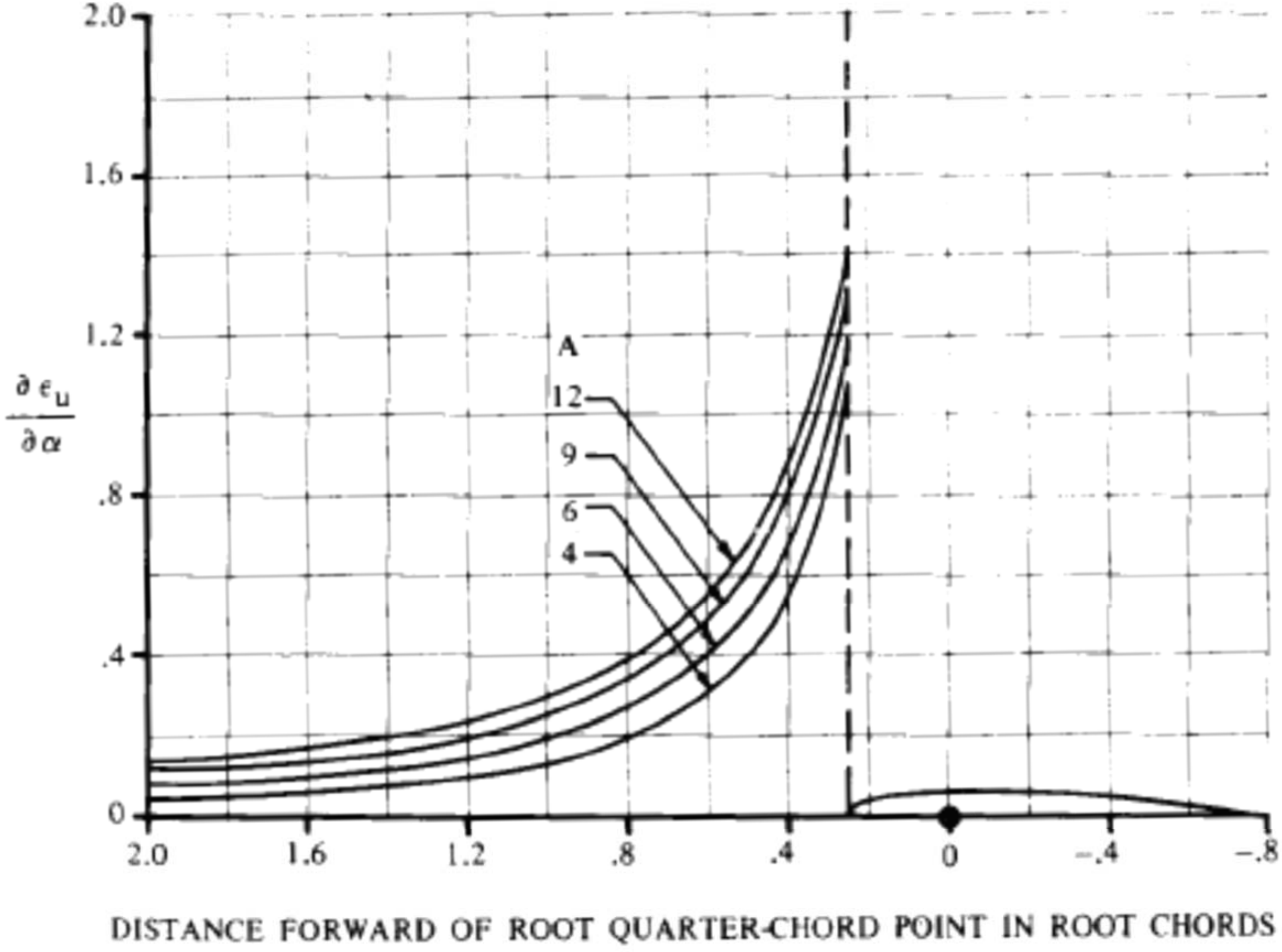


Fig. 16.11 Upwash estimation (subsonic only). (Ref. 37)

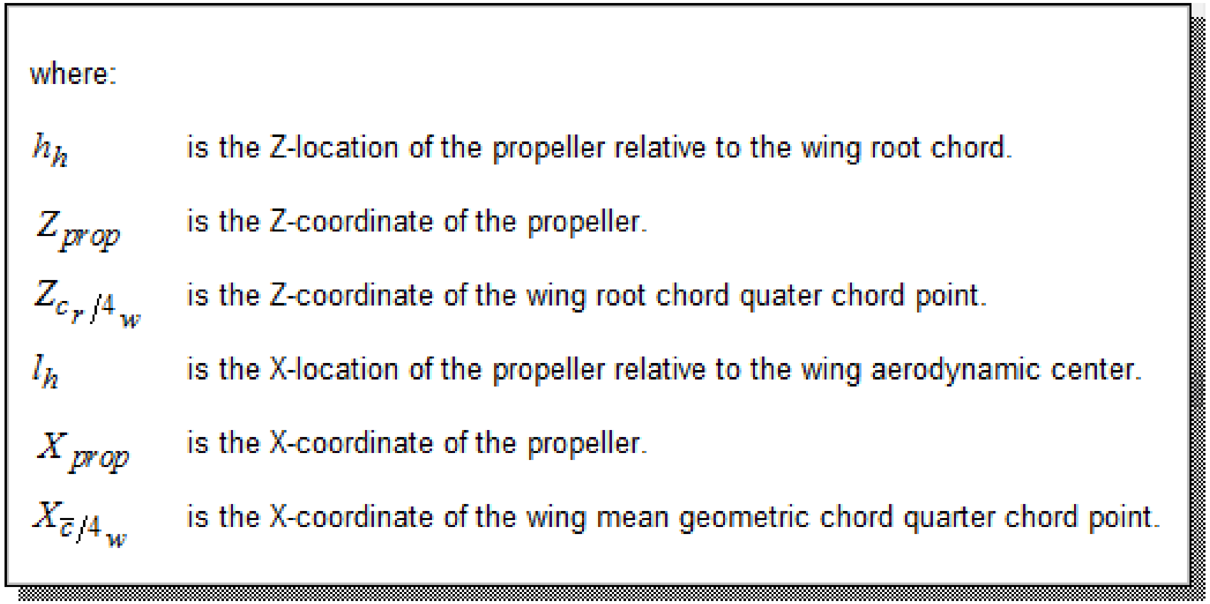
Propulsive Derivatives $C_{M_{T_\alpha}}$

For propeller driven airplanes:

For propeller behind the wing, the propeller downwash gradient is computed with the same method used to calculate horizontal tail downwash gradient with appropriate substitution

$$h_h = Z_{prop} - Z_{c_r/4_w}$$

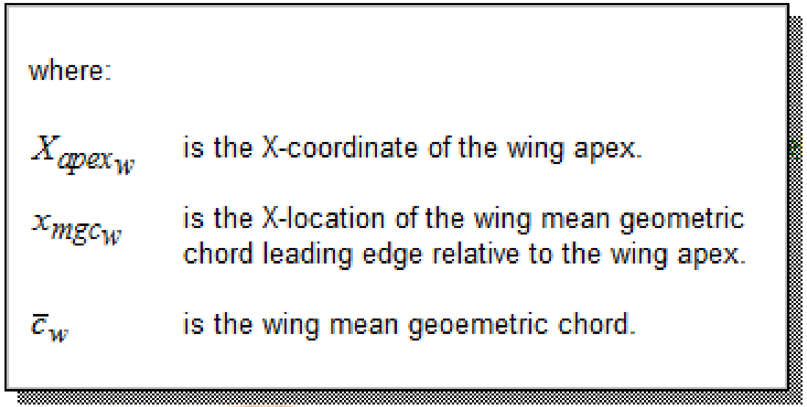
$$l_h = X_{prop} - X_{\bar{c}/4_w}$$



where:

h_h	is the Z-location of the propeller relative to the wing root chord.
Z_{prop}	is the Z-coordinate of the propeller.
$Z_{c_r/4_w}$	is the Z-coordinate of the wing root chord quater chord point.
l_h	is the X-location of the propeller relative to the wing aerodynamic center.
X_{prop}	is the X-coordinate of the propeller.
$X_{\bar{c}/4_w}$	is the X-coordinate of the wing mean geometric chord quarter chord point.

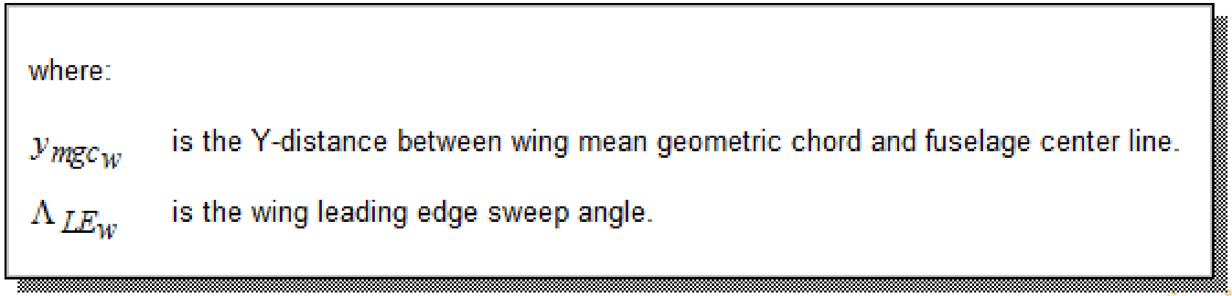
$$X_{\bar{c}/4_w} = X_{apex_w} + x_{mgc_w} + 0.25\bar{c}_w$$



where:

X_{apex_w}	is the X-coordinate of the wing apex.
x_{mgc_w}	is the X-location of the wing mean geometric chord leading edge relative to the wing apex.
$\bar{c}_w$	is the wing mean geoemetric chord.

$$x_{mgc_w} = y_{mgc_w} \tan \Lambda_{LE_w}$$



where:

y_{mgc_w}	is the Y-distance between wing mean geometric chord and fuselage center line.
Λ_{LE_w}	is the wing leading edge sweep angle.

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